DE–2788 **105**

DISTANCE EDUCATION

M.C.A. (N.S) DEGREE EXAMINATION, DECEMBER 2011.

DISCRETE MATHEMATICS

(2001 onwards)

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) Show that $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$. (10)

   (b) Obtain the principal disjunctive normal form of $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$. (10)

2. (a) Show that SVR is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$. (10)

   (b) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$. (10)

3. (a) Show by means of an example that $A \times B \neq B \times A$ and $(A \times B) \times C \neq A \times (B \times C)$. (10)

   (b) If $M_k = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $M_s = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ are the rational matrices, find $M_{R \circ S}, M_{\tilde{R}}, M_{\tilde{S}}, M_{R \tilde{\circ} S}$ and show that $M_{R \tilde{\circ} S} = M_{\tilde{S} \circ \tilde{R}}$. (10)

4. (a) If $f(x) = x + 2, g(x) = x - 2$ and $h(x) = 3x$ for $x \in R$, where R is the set of real numbers. Find $g \circ f$; $f \circ g$; $f \circ f$; $g \circ g$; $f \circ h$; $h \circ g$; $h \circ f$ and $f \circ h \circ g$. (10)

   (b) Show that $f : X \rightarrow Y$ is one-to-one iff any proper subsets of X are mapped into proper subsets of Y; that is if $A \subset B \subseteq X$, then $f(A) \subset f(B) \subseteq Y$. (10)

5. (a) Given the algebraic system $\langle N, + \rangle$ and $\langle Z_4, +4 \rangle$, where N is the set of natural numbers and + is the operation of addition on N, Show that there exists a homomorphism from $\langle N, + \rangle$ to $\langle Z_4, +4 \rangle$. (10)

   (b) For any commutative monoid $\langle M, * \rangle$, prove that the set of idempotent elements of M, forms a submonoid. (5)

   (c) Show that the set of all invertible elements of a monoid form a group under the same operation as that of the monoid. (5)

6. (a) Show that in a group $\langle G, * \rangle$, if for any $a, b \in G$, $(a * b)^2 = a^2 * b^2$, then $\langle G, * \rangle$ must be abelian. (6)

   (b) Show that the set of all elements a of a group $\langle G, * \rangle$ such that $a * x = x * a$ for every $x \in G$ is a subgroup of G. (7)

   (c) Prove that, the order of a subgroup of a finite group divides the order of the group. (7)

7. (a) State and prove the fundamental theorem of group homomorphism. (9)

   (b) Define field. (3)

(c) Prove that the ring homomorphism preserves the distributive property. (8)

8\. (a) Define the length of the path. (3)

(b) Show by means of an example that a simple diagraph in which exactly one node has indegree '0' and every other mode has indegree '1' is not necessarily a directed tree. (10)

(c) In a simple digraph $G,=\langle V, E\rangle$, prove that, every node of the digraph lies in exactly one strong component. (7)