(DMCA 108)

M.C.A.(Previous) DEGREE EXAMINATION, MAY 2006
PAPER - VIII - DISCRETE MATHEMATICS

Time: Three hours Maximum: 75 marks

SECTION - A - (3 X 15 = 45 marks)
Answer any THREE of the following

1. (a) Test the validity of the following argument.
   If I study, then I will not fail mathematics
   If I do not play basket ball, then I will study.
   But I failed Mathematics
   Therefore I must have played basket ball.
   (b) Find the truth tables for (i) $p \vee \neg q$ (ii) $\neg p \wedge \neg q$

2. (a) For any $n \in N$, Let $D_n = (0,1/n)$, the open interval from O to 1. Find :
   (i) $D_3 \cup D_7$ (ii) $D_3 \cap D_{20}$ (iii) $D_5 \cup D_t$ (iv) $D_5 \cap D_t$.
   (b) Let $f : R \rightarrow R$ be defined by $f(x) = 2x - 3$. Now *f* is One-to-one and Onto: hence *f* has an inverse function f^{-1}. Find a formula for f^{-1}.

3. (a) Show that every finite semigroup has an idempotent.
   (b) Let (*S*,*) be a semigroup and $Z \in S$ be a left zero. Show that for any $x \in S, x * Z$ is also a left zero.

4. (a) Explain the properties of binary operations.
   (b) Determine whether $H_1 = \{0,5,10\}$ and $H_2 = \{0,4,8,12\}$ are subgroups of Z_{15}.

5. Let $N = \{1,2,3,.....\}$ be ordered by divisibility. State whether each of the following subsets of *N* are linearly ordered. (a) $\{24,2,6\}$ (b) $N = \{1,2,3\}$ (c) $\{7\}$ (d) $\{3,15,5\}$ (e) $\{2,8,32,4\}$ (f) $\{15,5,30\}$

SECTION - B - (5 X 5 = 25 marks)
Answer any FIVE of the following

6. Negate each of the following statements:
   (a) If the teacher is absent, then some students do not complete their homework.
   (b) All the students completed their homework and the teacher is present.
   (c) Some of the students did not complete their homework or the teacher is absent.

7. Construct the truth table for $[(p \rightarrow q) \wedge \neg p] \rightarrow \neg q$.

8. Consider the group $G = \{1,2,3,4,5,6\}$ under multiplication modulo 7.
   (a) Find the multiplication table of G.
   (b) Find $2^{-1}, 3^{-1}, 6^{-1}$.
   (c) Find the orders and subgroups generated by 2 and 3.
   (d) Is G cyclic.

9. Define well-ordered sets with example.

10. Suppose $P(n) = a_0 + a_1 n + a_2 n^2 + ... + a_m n^m$ has degree m. Prove $P(n) = O(n^m)$.

11. Explain conjunction and disjunction normal forms.

12. Solve the recurrence relation an $a_n = a_{n-1} + 3n^2 + 3n + 1$ where $a_0 = 1$ by method of substitution.

13. Using arithmetic modulo m=15, evaluate:
    (a) 9 + 13
    (b) 7 + 11
    (c) 4 - 9
    (d) 2 - 10

SECTION - C - (5 X 1 = 5 marks)
Answer any ALL of the following

14. What is symmetric relation?
15. What is composition function?
16. What is Homomorphisms?
17. Define submonoid.
18. What is subalgebra?

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