(DMCA 108)

M.C.A. DEGREE EXAMINATION, DECEMBER 2006

First Year

Paper - VIII : DISCRETE MATHEMATICS

Time : Three hours Maximum : 75 marks

SECTION A - (3 X 15 = 45 marks)

Answer any THREE of the following.

1. (a) Consider the following sets:

$$\phi,\ A=\{1\}\quad B=\{1,3\}\quad C=\{1,5,9\}\quad D=\{1,2,3,4,5\}\quad E=\{1,3,5,7,9\}\quad U=\{1,2,\ldots.8,9\}$$

Insert the correct symbol $\subseteq$ or $\not\subset$ between each pair of sets.

(b) Construct the truth table for (P

2. (a) Determine types of relations with examples.

(b) Prove the set I of all real numbers between 0 and 1 is uncountable.

3. (a) Show that the set N of natural numbers is a semigroup under the operation $x * y = \max\{x, y\}$. Is it a monoid?

(b) Prove that for any commutative monoid $(m,*)$, the set of idempotent elements of M forms a sub monoid.

4. Prove that every row or column in the composition table of a group $(G,*)$ is a permutation of the elements of G.

5. (a) If $(G,*)$ is an abelian group, then for all $a, b \in G$ show that $(a*b)^n = a^n * b^n$.

(b) Show that if every element in a group is its own inverse, than the group must be abelian.

SECTION B - (5 X 5 = 25 marks)

Answer any FIVE of the following.

6. Explain Hasse Diagram.

7. Show that the proportions $\neg(p \cap q)$ and $\neg p \vee \neg q$ are logically equivalent.

8. Consider the relation $R = \{(1,1)(1,2)(1,3)(3,3)\}$ on the set $A = \{1,2,3\}$. Determine whether or not the relation R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Anti symmetric

9. Find the domain D of each of the following real-valued functions.

(a) $f(x) = \frac{1}{x-2}$

(b) $f(x) = x^2 - 3x - 4$

(c) $\sqrt{25 - x^2}$

10. Let $V = \{1,2,3,4\}$ and let $f = \{(1,3)(2,1)(3,4)(4,3)\}$ and $g = \{(1,2)(2,3)(3,1)(4,1)\}$ and

(a) fog

(b) gof

(c) fof

11. Let $A = (1,2,3,4,5)$. Determine the truth value of each of the following statements.

(a) $(\exists x \in A)(x + 3 = 10)$

(b) $(\forall x \in A)(x + 3 < 10)$

(c) $(\exists x \in A)(x + 3 < 5)$

(d) $(\forall x \in A)(x + 3 \leq 7)$

12. Let S be a semigroup with identity e, and let b and b^1 be inverses of a. Such that $b=b^1$

13. Explain Lagrange's Theorem.

SECTION C - (5 X 1 = 5 marks)

Answer ALL of the following:

14. What is a function?

15. What is structure?

16. Consider the ring $Z_{10} = \{0,1,2,......9\}$ of integers modulo 10. Then find the units of Z_{10}.

17. Rewrite the following statement using the conditional. If it is cold, he wears a hat.

18. Define semigroup homomorphism.