(DSDM 11)

B.Sc. DEGREE EXAMINATION, MAY 2006
(Examination at the end of First Year)
Part II - Information Technology
Paper I : DISCRETE MATHEMATICS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.
All questions carry equal marks.

1. (a) Using mathematical induction prove that for all positive integers *n*.

$6^{n+2} + 7^{2n+1}$ is divisible by 43.

(b) Prove that proposition P that the sum of first *n* positive integers is $\frac{1}{2}n(n+1)$: that is

$$P(n) = 1+2+3+...+n = \frac{1}{2}n(n+1)$$

2. (a) Let R and S be the following relations on $A = \{1,2,3\}$; $R = \{(1,1)(1,2)(2,3)(3,1)(3,3)\}$;

$S = \{(1,2)(1,3)(2,1)(3,3)\}$. Find (i) $R \cap S, R \cup S, R^C$ (ii) $R \circ S$ (iii) $S^2 = S \circ S$.

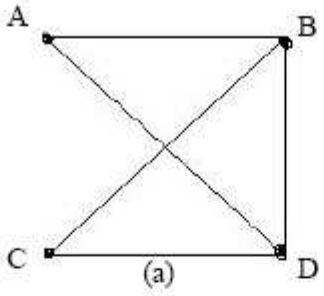
(b) Consider the set Z of integers. Design a R b by $b = a^r$ for some positive integer *r*. . Show that R is a partial order on Z, that is, show that R is (i) Reflexive (ii) Antisymmetric (iii) Transitive.

3. (a) Prove that any planar graph is 4-colorable.

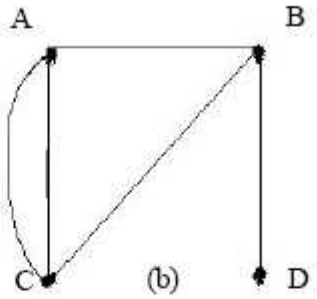
(b) Draw the the graph G corresponding to given adjacency matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

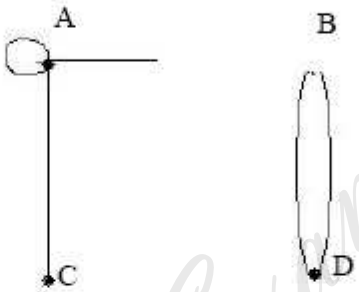
4. (a) Find the adjacency matrix A of each graph.



(a)



(b)



(b) (i) Find the number of connected graphs with four vertices and draw them.
(ii) Draw all trees with five or fewer vertices.

5. Draw all possible non similar trees T where
(a) T is a binary tree with three nodes.
(b) T is a 2-tree with four external nodes.

6. (a) Prove each of the following statements.
(i) Any integer a is of the form $3k, 3k+1, 3k+2$.
(ii) One of three consecutive integers in a multiple of 3.

(b) Write the dual of each Boolean equation.

(i) $(a*1)*(0+a')=0$

(ii) $a+a'b=a+b$

7. Find the prime implicants and a minimal sum-of-products form for each of the following complete sum-of-products Boolean expressions.

(a) $E_1 = xyz + xyz' + x'yz' + x'y'z$

(b) $E_2 = xyz + xyz' + xy'z + x'yz + x'y'z$

(c) $E_3 = xyz + xyz' + x'yz' + x'y'z' + x'y'z$

8. (a) State and prove Euler's formula for planar graphs.
   (b) Write a short notes on fuzzy sets and possibility theory.
9. Construct a binary tree for the following expression.

$$(a+5)X[(3b+c)/(d+2)]$$

10. (a) Prove that a binary tree with n nodes has exactly (n+1) null branches.
    (b) Formulate an algorithm for the inorder traversal of a binary tree.

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