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ANNA UNIVERSITY OF TECHNOLOGY, COIMBATORE  
B.E. / B.TECH. DEGREE EXAMINATIONS : NOV / DEC 2010

REGULATIONS : 2008

THIRD SEMESTER

080100008 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(COMMON TO CIVIL / EEE / EIE / ICE / ECE / BIOMEDICAL / BIOTECH /

AERO / AUTO / CSE / IT / MECHANICAL / CHEMICAL / FT / TT / TC )

Time: 3 Hours

Max.Marks: 100

PART - A

(20 x 2 = 40 Marks)

ANSWER ALL QUESTIONS

1. State the conditions for f(x) to have Fourier series expansion.
2. Write  $a_0, a_n$  in the expansion of  $x+x^3$  as Fourier Series in  $(-\pi, \pi)$ .
3. Expand  $f(x)=1$  in a sine series in  $0 < x < \pi$
4. Find Root Mean Square value of the function  $f(x) = x$  in the interval  $(0, l)$ .
5. Define Fourier Transform Pair.
6. Find Fourier Cosine transform of  $e^{-2x}$ .
7. If  $F(S)$  is the Fourier Transform of  $f(x)$ , show that the Fourier Transform of  $e^{iax} f(x)$  is  $F(S+a)$ .
8. State Parseval's Identity for Fourier Transform.
9. Eliminate the arbitrary constants a & b from  $z = (x^2 + a)(y^2 + b)$ .
10. Form the PDE by eliminating the functions from  $z = f(x+t) + g(x-t)$ .
11. Find the complete integral  $q = 2px$ .
12. Solve  $(D^3 - 3DD'^2 + 2D'^3)z = 0$ .

13. Find the nature of PDE  $4u_{xx} + 4u_{xy} + u_{yy} + 2u_x - u_y = 0$ .
14. What are the various solutions of one dimensional Wave Equation?
15. A string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string into the form  $y = y_0 \sin(\frac{\pi x}{l})$  from which it is released at time  $t=0$ .  
Formulate this problem as the boundary value problem.
16. A rod of length 20cm whose one end is kept at  $30^\circ\text{C}$  and the other end is kept at  $70^\circ\text{C}$  is maintained so until steady state prevails. Find the steady state temperature.
17. Find  $Z[e^{-an}]$ .
18. Prove that  $Z[n] = \frac{z}{(z-1)^2}$
19. Prove that  $Z[f(n+1)] = zF(z) - zf(0)$
20. State Initial and Final value theorem on Z- transform.

PART - B

(5 x 12 = 60 Marks)

ANSWER ANY FIVE QUESTIONS

21(a). If  $f(x) = \left(\frac{\pi-x}{2}\right)$  find the Fourier Series of the period  $2\pi$  in the interval  $(0, 2\pi)$ .

Hence deduce that  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$  (8)

(b). Find the Fourier expansion of  $f(x) = x$  in the interval  $(-\pi, \pi)$  (4)

22. Show that the Fourier Transform of  $f(x) = \begin{cases} a^2 - x^2 & |x| \leq a \\ 0 & \text{otherwise} \end{cases}$  is

$$2\sqrt{\frac{2}{\pi}} \left( \frac{\sin as - as \cos as}{s^3} \right)$$

Hence deduce that  $\int_0^{\infty} \frac{\sin t - t \cos t}{t^3} dt = \frac{\pi}{4}$

Using Parseval's Identity show that  $\int_0^{\infty} \left( \frac{\sin t - t \cos t}{t^3} \right)^2 dt = \frac{\pi}{15}$

x	0	1	2	3	4	5	6
f(x)	9	18	24	28	26	20	9

23.(a) Solve  $(mz-ny)p + (nx-lz)q = ly-mx$  (6)

(b) Solve  $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y$  (6)

24. A string of length  $l$  is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity  $V = \begin{cases} cx & , 0 \leq x \leq l/2 \\ c(l-x) & , l/2 \leq x \leq l \end{cases}$   
Find the displacement function  $y(x,t)$ .

25 (a) Evaluate  $z^{-1} \left[ \frac{z}{z^2 + 7z + 10} \right]$  (6)

(b) Using z-transforms solve  $u(n+2) - 5u(n+1) + 6u(n) = 4^n$  given that  $u(0)=0, u(1)=1$  (6)

26(a) Find the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of,  $y=f(x)$  as given in the following table:- (6)

26(b) Find the Fourier transform of  $f(x) = \begin{cases} 1-|x| & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$

hence find the value of  $\int_0^{\infty} \frac{\sin^4 x}{x^4} dx$  (6)

27. A metal bar 30cm long has its ends A and B kept at 20°C and 80°C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature  $u(x,t)$  taking  $x=0$  at A.

28(a) Solve  $p(1+q) = qz$  (6)

(b) Using Convolution theorem, evaluate  $Z^{-1} \left[ \frac{z^2}{(z-1)(z-3)} \right]$  (6)

\*\*\*\*\*THE END\*\*\*\*\*