ANNA UNIVERSITY OF TECHNOLOGY, COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : NOV / DEC 2011

REGULATIONS : 2008

FIFTH SEMESTER : CSE

080230017 - DISCRETE MATHEMATICS

Time: 3 Hours **Max . Marks : 100**

PART - A

(10 x 2 = 20 Marks)

ANSWER ALL QUESTIONS

1. Construct the truth table for $(p \vee q) \rightarrow (p \wedge q)$
2. Show that the qV(pVq)V(pΛq)statement is a tautology.
3. Symbolise the statement “All men are giants”
4. Define quantifiers
5. If A and B are finite sets show that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
6. Show that in any Boolean Algebra $(a+b)(a'+c) = ac + a' + bc$
7. Define a characteristic function of a set
8. State whether the function f(x) = $5x^2 + 7$ is injection, surjection or bijection on R, the set of real numbers.
9. Define a normal subgroup of a group.
10. If the minimum distance between two code words is 7, then find how many errors can be detected and how many errors can be corrected?

PART - B

(5 x16 = 80 MARKS)

ANSWER ALL QUESTIONS

11a) (i) Find pdnf of $P \vee (P \rightarrow (Q \vee (Q \rightarrow R)))$ without using truth table.

(ii) Use the indirert method to show that $r \rightarrow 7q,\ r \vee s,\ s \rightarrow 7q,\ p \rightarrow q \Rightarrow 7p$.

(OR)

11b) (i) Construct the truth table for $((p \vee q) \wedge ((p \rightarrow r) \wedge (q \rightarrow r))) \rightarrow r$

(ii) Using direct method prove $(p \rightarrow q) \rightarrow r, p \wedge s, q \wedge t \Rightarrow r$

12a) (i) Prove that (∃x) (P(x)∧S(x)) , (∀x)(P(x) → R(x)) ⇒(∃x) (R(x)∧S(x))

(ii) By indirect method, prove that (∀x) (P(x) ∨ Q(x)) ⇒ (∀x) (P(x) ∨ (∃x) Q(x).

(OR)

12b) (i) Prove that (∃x) M(x) follows logically from the premises (∀x)(H(x) → M(x)) and (∃x) H(x).

(ii) Prove the following implication

$\forall x(P(x) \rightarrow Q(x) \wedge \forall x(Q(x) \rightarrow R(x) \Rightarrow \forall x(P(x) \rightarrow R(x)$

13a) (i) If R is the relation on the set of positive integers such that (a, b) ∈ R if and only if a^2 + a is even, prove that R is an equivalence relation.

(ii) If {L,≤} is a Lattice, then for any $a, b, c \in L$ prove that

$a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(OR)

13b) (i) Define the relation P on $\{1,2,3,4\}$ by $P = \{(a,b) / |a-b| = 1\}$. Determine the adjacency matrix of P^2

(ii) Simplify the Boolean expression $\left((x_1 + x_2) + (x_1 + x_3)\right) x_1 . \overline{x_2}$

14a) (i) Let $f : R \rightarrow R$ and $g : R \rightarrow R$ where R is the set of real numbers, find $f \bullet g$ and $g \bullet f$, if $f(x) = x^2 - 2$ and $g(x) = x + 4$

(ii) Show that the function $f(x, y) == x^y$ is a primitive recursive function.

(OR)

14b) (i) Show that $f : R-\{3\} \rightarrow R-\{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is a bijection

(ii) Using characteristic function show that $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$

15a) (i) If H is a subgroup of g such that $x^2 \in$ H for every x $\in$ G prove that H is a normal subgroup of G

(ii) Find the minimum distance of the encoding function e = $B^2 \rightarrow B^4$ given by e(00) = 0000, e(10) = 0110, e(01) = 1011, e(11) = 1100

(OR)

15b) (i) If (G, $\star$) is an abelian group then for all a, b $\in$ G, show that if $(a\star b)^n = a^n \star b^n$.

(ii) Prove that a code can correct all combinations of k or fewer errors if and only if the minimum distance between any two code word is atleast 2k+1

*****THE END*****