

Code No. 35

Total No. of Questions : 40]

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MATHEMATICS

Instructions : i) The question paper has five **Parts - A, B, C, D and E**.
Answer all the parts.

ii) **Part - A** carries 10 marks, **Part - B** carries 20 marks,
Part - C carries 40 marks, **Part - D** carries 20 marks and
Part - E carries 10 marks.

PART - A

Answer all the ten questions :

$10 \times 1 = 10$

1. Find the least positive remainder when 7^{30} is divided by 5.
2. If $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, find x .
3. Define a subgroup.
4. Find the direction cosine of the vector $2\hat{i} - 3\hat{j} + 2\hat{k}$.
5. If the radius of the circle $x^2 + y^2 + 4x - 2y - k = 0$ is 4 units, then find k .
6. Find the equation of the parabola if its focus is (2, 3) and vertex is (4, 3).
7. Find the value of $\sin \left[\frac{1}{2} \cos^{-1} (-1) \right]$.

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8. If $1, \omega, \omega^2$ are the cube roots of unity, find the value of $(1 - \omega + \omega^2)^6$.
9. Differentiate $3^x \sinh x$ w.r.t. x .
10. Integrate $\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ w.r.t. x .

PART - B

Answer any ten questions :

$10 \times 2 = 20$

11. If $a \equiv b \pmod{m}$ and n is a positive divisor of m , prove that

$$a \equiv b \pmod{n}.$$

12. Without actual expansion show that
$$\begin{vmatrix} 43 & 1 & 6 \\ 35 & 7 & 4 \\ 17 & 3 & 2 \end{vmatrix} = 0.$$

13. Is $G = \{0, 1, 2, 3\}$, under \otimes modulo 4 a group? Give reason.

14. Find the equation of two circles whose diameters are $x + y = 6$ and $x + 2y = 4$ and whose radius is 10 units.

15. Find the area of the parallelogram whose diagonals are given by the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$.

Find the eccentricity of the ellipse ($a > b$), if the distance between the directrices is 5 and distance between the foci is 4.

Solve $\cot^{-1} x + 2 \tan^{-1} x = \frac{5\pi}{6}$.

Find the least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$.

If $y = (x + \sqrt{1+x^2})^m$, prove that $(\sqrt{1+x^2}) \frac{dy}{dx} - my = 0$.

Show that for the curve $y = be^{\frac{x}{a}}$ the subnormal varies as the square of the ordinate y .

1. Evaluate $\int_1^e \log_e x \, dx$.

2. Find the order and degree of the differential equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}.$$

PART - C

I. Answer any three questions :

$$3 \times 5 = 15$$

23. Find the G.C.D. of $a = 495$ and $b = 675$ using Euclid Algorithm.

Express it in the form $495(x) + 675(y)$. Also show that x and y

are not unique where $x, y \in \mathbb{Z}$.

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24. Solve the linear equations by matrix method :

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

25. a) On the set of rational numbers, binary operation $*$ is defined by $a * b = \sqrt{a^2 + b^2}$, $a, b \in R$, show that $*$ is commutative and associative. Also find the identity element.

b) If a is an element of the group $(G, *)$, then prove that

$$(a^{-1})^{-1} = a.$$

26. a) Find the sine of the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} + \hat{k}$.

b) Show that the vectors $\hat{j} + 2\hat{k}$, $\hat{i} - 3\hat{j} - 2\hat{k}$ and $-\hat{i} + 2\hat{j}$ form the vertices of the vectors of an isosceles triangle.

II. Answer any two questions :

2 × 5 = 10

27. a) Derive the condition for the two circles

$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0 \text{ and}$$

$$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \text{ to cut orthogonally.} \quad 3$$

- b) Show that the radical axis of the two circles

$$2x^2 + 2y^2 + 2x - 3y + 1 = 0 \text{ and}$$

$x^2 + y^2 - 3x + y + 2 = 0$ is perpendicular to the line joining the centres of the circles. 2

28. a) Find the ends of latus rectum and directrix of the parabola $y^2 - 4y - 10x + 14 = 0$. 3

- b) Find the value of k such that the line $x - 2y + k = 0$ be a tangent to the ellipse $x^2 + 2y^2 = 12$. 2

29. a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z - xyz = 0$. 3

- b) Find the general solution of $\tan 4\theta = \cot 2\theta$. 2

III. Answer any three of the following questions : 3 \times 5 = 15

30. a) Differentiate $\tan x$ w.r.t. x from the first principle. 3

- b) If $y = \tan^{-1} \left[\frac{2 + 3x^2}{3 - 2x^2} \right]$, prove that $\frac{dy}{dx} = \frac{2x}{1 + x^4}$. 2

31. a) If $y = \cos (p \sin^{-1} x)$, prove that

$$(1 - x^2) y_2 - x y_1 + p^2 y = 0. \quad \text{3}$$

- b) Find the equation of the normal to the curve $y = x^2 + 7x - 2$ at the point where it crosses y -axis. 2

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32. a) Integrate $e^{3x} \left(\frac{3 + \tan x}{\cos x} \right)$ w.r.t. x . 3

b) Find the angle between the curves $4y = x^3$ and $y = 6 - x^2$ at $(2, 2)$. 2

33. a) If $x^m y^n = (x + y)^{m+n}$, prove that $x \frac{dy}{dx} = y$. 3

b) Integrate $\frac{1}{7 - 6x - x^2}$ w.r.t. x . 2

34. Find the area between the curves $y^2 = 6x$ and $x^2 = 6y$. 5

PART - D

Answer any two of the following questions :

$2 \times 10 = 20$

35. a) Define hyperbola as a locus and hence derive the equation of the hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. 6

b) Show that
$$\begin{vmatrix} b^2 + c^2 & ab & ac \\ ba & c^2 + a^2 & bc \\ ca & cb & a^2 + b^2 \end{vmatrix} = 4a^2 b^2 c^2$$
. 4

36. a) If $\cos \alpha + 2 \cos \beta + 3 \cos \gamma = 0$, $\sin \alpha + 2 \sin \beta + 3 \sin \gamma = 0$,

show that i) $\cos 3\alpha + 8 \cos 3\beta + 27 \cos 3\gamma = 18 \cos (\alpha + \beta + \gamma)$

ii) $\sin 3\alpha + 8 \sin 3\beta + 27 \sin 3\gamma = 18 \sin (\alpha + \beta + \gamma)$. 6

b) Prove that $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2 [\vec{a} \quad \vec{b} \quad \vec{c}]$. 4

- a) The volume of a sphere is increasing at the rate of 4π c.c./sec. Find the rate of increase of the radius and its surface area when the volume of the sphere is 288π c.c. 6

- b) Find the general solution of $\sqrt{3} \tan x = \sqrt{2} \sec x - 1$. 4

- a) Show that $\int_0^{\pi/4} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$. 6

- b) Solve the differential equation

$$\tan y \frac{dy}{dx} = \sin(x + y) + \sin(x - y). \quad 4$$

PART - E

Answer any one of the following questions :

$1 \times 10 = 10$

9. a) Find the cube roots of $3 - i\sqrt{3}$ and find their continued product. 4

- b) Show that $(\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$. 4

- c) Find the length of the chord of the circle

$$x^2 + y^2 - 6x - 2y + 5 = 0 \text{ intercepted by the line } x - y + 1 = 0. \quad 2$$

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40. a) Evaluate $\int_0^3 \frac{\sqrt{x+2}}{\sqrt{x+2} + \sqrt{5-x}} dx.$ 4

b) Show that among all the rectangles of a given perimeter, the square has maximum area. 4

c) Differentiate $\sec(5x)^0$ w.r.t. $x.$ 2

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