North Maharashtra University, Jalgaon
Question Bank

(New syllabus w.e.f. June 2007)

Class: F. Y. B. Sc.

Subject: Mathematics

Paper II

(CALCULUS)

Prepared By:-

Prof. R. B. Patel	**Art, Science & Comm. College, Shahada**
Dr. B. R. Ahirrao	**Jaihind College, Dhule**
Prof. S. M. Patil	**Art, Science & Comm. College, Muktainagar**
Prof. A. S. Patil	**Art, Science & Comm. College, Navapur**
Prof. G. S. Patil	**Art, Science & Comm. College, Shahada**
Prof. A. D. Borse	**Jijamata College, Nandurbar**

Unit I

Limit, Continuity, Differentiability and Mean Value Theorem

Q.1 Objective Questions **Marks – 02**

1. $\lim_{x\to 5}\frac{x^2-4x-5}{x^2+2x-35}$ *is equal to*

a) 1 b) $\frac{1}{2}$ c) $\frac{-1}{2}$ d) none of these

2. $\lim_{x\to 1}\frac{\cos x}{x-1}$ *is equal to*

a) 0 b) 1 c) -1 d) none of these

3. Evaluate $\lim_{x\to 0}\frac{x-\tan x}{x^3}$

a) $\frac{-1}{3}$ b) $\frac{1}{3}$ c) 0 d) 1

4. The value of the $\lim_{x\to 0}\frac{\log(\sin 2x)}{\log(\sin x)}$ is

a) 2 b) 0 c)1 d) -1

5. $\lim_{x\to\infty}\frac{x^n}{e^x}$ is equal to

a) 1 b) -1 c) 2 d) 0

6. $\lim_{x\to 0}\frac{\log(\sin ax)}{\log(\sin bx)}$ $,(a,b>0)$ is equal to

a) -1 b) 1 c) 0 d) none of these

7. $\lim_{x\to 0} x\log x$ is equal to

a) 0 b) 1 c) 2 d) -1

8. $\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{\sin x}\right]$ is equal to

a) 0 b)1 c) -1 d) none of these

9. $\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{e^x-1}\right]$ is equal to

a) 1 b) $\frac{-1}{2}$ c) $\frac{1}{2}$ d) 0

10. $\lim_{x\to 0} x^x$ is equal to

a) 1 b) -1 c) 2 d) none of these

11. $\lim_{x\to \pi/4} (\tan x)^{\tan 2x}$ is

a) e b) $\frac{1}{e}$ c) $\frac{-1}{e}$ d) – e

12. The function $f(x)=x\sin\frac{1}{x}$, *for* $x\neq 0$ *and*
$f(0)=0$, *for* $x=0$

a) Continuous and derivable

b) Not continuous but derivable

c) Continuous but not derivable

d) Neither continuous nor derivable at the point x = 0

13. The function $f(x)=x^2\sin\frac{1}{x}$, *for* $x\neq 0$ *and*
$f(0)=0$, *for* $x=0$ is

a) Continuous and derivable

b) Not continuous but derivable

c) Continuous but not derivable

d) Neither continuous nor derivable

14. For which value of $c\in(a,b)$, the Roll's theorem is verified for the function

$f(x)=\log\left[\frac{x^2+ab}{x(a+b)}\right]$ *defined on* $[a,b]$

a) Arithmetic mean of a & b b) Geometric mean of a & b

c) Harmonic mean of a & b d) None of these .

15. For which value of $c \in (a,b) = (0, 2\pi)$, the Rolle's theorem is applicable for the function $f(x) = \sin x \quad , in[0, 2\pi]$

a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{3}$

16. For which value of $c \in \left(0, \frac{\pi}{2}\right)$, the Rolle's theorem is applicable for the function $f(x) = \sin x + \cos x \quad in\left[0, \frac{\pi}{2}\right]$

a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

17. For which value of $c \in (1,5)$, the Rolle's theorem is verified for the function $f(x) = x^2 - 6x + 5 \ in[1,5]$

a) 1 b) 2 c) 3 d) 4

18. for which value of c∈ (-2, 3) . the L.M.V.T. is verified for the function $f(x) = x^2 - 3x + 2 \quad in[-2, 3]$

a) 1 b) $\frac{1}{2}$ c) $\frac{-1}{2}$ d) 0

19. L.M.V.T is verified for the function $f(x) = 2x^2 - 7x + 10 \quad in[2,5]$

a) $\frac{5}{2}$ b) $\frac{1}{2}$ c) 0 d) $\frac{7}{2}$

20. For which value of $c \in \left(0, \frac{\pi}{2}\right)$ C.M .V.T. is applicable for the function

f(x) = sin x , g(x) = cos x in [0, π/2]

a) 0 b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

21. If the C.M.V.T. is applicable for the function

f(x) = e^x , g(x) = e^{-x} , in [a, b] find the value of $c \in (a,b)$

a) $\frac{a+b}{2}$ b) $\sqrt{ab}$ c) a + b d) none of these

22. If the C.M.V.T. is applicable for the function

$f(x) = 1/x^2$, $g(x) = 1/x$, in [a, b] find the value of C.

a) $\frac{a+b}{2}$ b) $\sqrt{ab}$ c) $\frac{2ab}{a+b}$ d) none of these

23. If $f(x) = \frac{\log x - \log 5}{x-5}$, $x \neq 5$ is continuous at x = 5 then find f(5)

a) 5 b) -5 c) $\frac{1}{5}$ d) $\frac{-1}{5}$

24. If $f(x) = \frac{1-\sin x}{(\pi - 2x)^2}$, $x \neq \frac{\pi}{2}$ is continuous at $x = \frac{\pi}{2}$ then f(π/2) is

a) $\frac{1}{8}$ b) $\frac{2}{3}$ c) 1 d) -1

25. If $f(x) = \frac{1-\cos x}{\sin x}$, $x \neq 0$ is continuous at x = 0 then value of f(0) is

a) 0 b) 1 c) -1 d) none of these

26. I f $f(x) = \frac{a^x - a^a}{a-x}$, $x \neq a$ is continuous at x = a , then find f(a)

a) $a^a \log a$ b) $-a^a \log a$ c) log a d) none of these

27. Evaluate $\lim_{x \to 0} \sin x \log x$

a) 0 b) 1 c) -1 d) $\frac{\pi}{2}$

28. Evaluate $\lim_{x \to 0} \tan x \log x$.

a) 0 b) 1 c)-1 d) none of these

29. $\lim_{x \to 1} \left[\frac{1}{\log x} - \frac{1}{x-1} \right]$ is equal to

a) $\frac{-1}{2}$ b) $\frac{1}{2}$ c) 2 d)-2

30. $\lim_{x \to 0} \left(\frac{1}{x} \right)^{1-\cos x}$ is equal to

a) -1 b) 1 c) 2 d) $\frac{1}{2}$

31. Evaluate $\lim_{x\to a}(x-a)^{(x-a)}$

a) $\frac{-1}{2}$ b) $\frac{1}{2}$ c) 1 d) -1

32. Evaluate $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{1}{\log(x-1)}\right)$

a) $\frac{-1}{2}$ b) $\frac{1}{2}$ c) 1 d) -1

33. Evaluate $\lim_{x\to\infty}(1+x)^{1/x}$

a) -1 b) 2 c) -2 d) 1

34. If $f(x)=\frac{\sin 4(x-3)}{x^2-2}$ $, x\neq 3$ is continuous at point x = 3 find f(3)

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{3}{2}$ d) none of these

35. Evaluate $\lim_{x\to 0}\frac{e^x-e^{\sin x}}{x-\sin x}$

a) 1 b) -1 c) 2 d) -2

Q.2 Examples **Marks – 04**

1. Evaluate $\lim_{x\to 0}\dfrac{\tan x - x}{x-\sin x}$

2. Evaluate $\lim_{x\to 0}\dfrac{e^x - 1 - x}{\log(1+x)-x}$

3. Evaluate $\lim_{x\to 0}\dfrac{\log(\tan 2x)}{\log(\tan x)}$

4. Evaluate $\lim_{x\to 0}\left(\dfrac{1}{x^2}-\cot^2 x\right)$

5. Evaluate $\lim_{x\to a}\left(2-\dfrac{x}{a}\right)^{\tan\left(\pi x/2a\right)}$

6. Evaluate $\lim_{x\to 0}(\cot x)^x \qquad , x>0$

7. Evaluate $\lim_{x\to 0}(\cot x)^{1/\log x}$

8. Evaluate $\lim_{x\to \infty}\left[\dfrac{\pi}{2}-\tan^{-1} x\right]^{1/x}$

9. Examine for continuity, the function

$$\begin{aligned} f(x) &= \frac{x^2}{a}-a, && for \quad 0<x<a \\ &= 0 \ , && for x=0 \\ &= a-\frac{a^3}{x^2} \ , && for x>a \end{aligned}$$

10. Using $\in - \delta$ definition , prove that

$$\begin{aligned} f(x) &= x^2\cos\frac{1}{x} \ , && if \ x\neq 0 \\ &= 0 \ , && if \ x=0 \end{aligned}$$ is continuous at x = 0

11. Examine the continuity of the function

$$\begin{aligned} f(x) &= \frac{e^{1/x}-1}{e^{1/x}+1}, && if \ x\neq 0 \\ &= 0 \ , && if \ x=0 \end{aligned}$$ at the point x = 0.

12. Examine the continuity of the function

$$f(x) = \frac{x^2 - 9}{x - 3}, \quad for \ \ 0 \le x < 3$$
$$= 6 \ , \quad for \ \ x = 3$$
$$= 8 - \frac{18}{x^2}, \quad for \ \ x > 3$$

at the point x = 3.

13. Examine the continuity of the function

$$f(x) = \frac{x^2}{4} - 4, \quad for \ \ 0 < x < 4$$
$$= 2 \ , \quad for \ \ x = 4$$
$$= 4 - \frac{64}{x^2}, \quad for \ \ x > 4$$

at the point x = 4..

14. If the function

$$f(x) = \frac{\sin 4x}{5x} + a, \quad for \ \ x > 0$$
$$= x + 4 - b \ , \quad for \ \ x < 0$$
$$= 1 \ , \quad for \ \ x = 0$$

is continuous at x = 0 , then find the values of a & b.

15. If f(x) is continuous on $[-\pi, \pi]$

$$f(x) = -2\sin x, \quad for \ \ -\pi \le x \le \frac{-\pi}{2}$$
$$= \alpha \sin x + \beta \ , \quad for \ \ \frac{-\pi}{2} < x < \frac{\pi}{2}$$
$$= \cos x, \quad for \ \ \frac{\pi}{2} \le x \le \pi$$

$Find \ \ \alpha \ \& \ \beta.$

16. Define differentiability of a function at a point and show that $f(x) = |x|$ is continuous, but not derivable at the point x = 0.

17. Discuss the applicability of Rolle's Theorem for the function $f(x) = (x-a)^m (x-b)^n \ \ defind \ in [a, b]$ where m, n are positive integers.

18. Discuss the applicability o Rolle's Theorem for the function

$f(x)=e^x(\sin x-\cos x) \quad in \left[\frac{\pi}{4},\frac{5\pi}{4}\right]$.

19. Verify Langrange's Mean Value theorem for the function

$f(x)=(x-1)(x-2)(x-3)$ defined in the interval $[0,4]$.

20. Find θ that appears in the conclusion of Langrange's Mean Value theorem

for the function $f(x)=x^3, a=1, h=\frac{1}{3}$.

21. Show that $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}, \quad if\, 0<a<b$.

And hence deduce that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\left(\frac{4}{3}\right)<\frac{\pi}{4}+\frac{1}{6}$

22. For 0 < a < b , Prove that $1-\frac{a}{b}<\log\frac{b}{a}<\frac{b}{a}-1$ and hence show that

$\frac{1}{6}<\log\frac{6}{5}<\frac{1}{5}$

23. If < a < b <1 , then prove that $\frac{b-a}{\sqrt{1-a^2}}<\sin^{-1}b-\sin^{-1}a<\frac{b-a}{\sqrt{1-b^2}}$

Hence show that $\frac{\pi}{6}-\frac{1}{2\sqrt{3}}<\sin^{-1}\frac{1}{4}<\frac{\pi}{6}-\frac{1}{\sqrt{15}}$

24. Show that $\frac{x}{1+x^2}<\tan^{-1}x<x, \quad x>0$

25. For x > 0 , prove that $x-\frac{x^2}{2}<\log(1+x)<x-\frac{x^2}{2(1+x)}$

26. Separate the interval in which $f(x)=x^3+8x^2+5x-2$ is increasing or decreasing.

27. Show that $\frac{x}{1+x}<\log(1+x)<x, \quad \forall x>0$

28. Show that $\frac{1}{1+x^2}<\frac{\tan^{-1}x}{x}<1, \quad \forall x>0$

29. With the help of Langrange's formula Prove that

$\frac{\alpha-\beta}{\cos^2\beta}<\tan\alpha-\tan\beta<\frac{\alpha-\beta}{\cos^2\alpha}, \quad where\, 0\le\beta\le\alpha\le\frac{\pi}{2}$

30. Verify Cauchy's Mean Value theorem for the function

f (x) = sinx, g (x) = cosx in $0 \le x \le \frac{\pi}{2}$

31. Show that $\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta,$ *where* $0 < \alpha < \theta < \beta < \frac{\pi}{2}$

32. If $f(x) = \frac{1}{x^2}$ *and* $g(x) = \frac{1}{x}$ in Cauchy's Mean Value Theorem, Show that C is the harmonic mean between a & b.

33. Discuss applicability of Cauchy's Mean Value Theorem for the function f (x) = sinx and g (x) = cosx in $[a,b]$.

34. Verify Cauchy's Mean value theorem $f(x) = \sqrt{x}, g(x) = \frac{1}{\sqrt{x}}$ $in[a,b]$

35. Find $c \in (0,9)$ such that

$$\frac{f(9) - f(0)}{g(9) - g(0)} = \frac{f'(C)}{g'(c)}$$ where $f(x) = x^3$ *and* $g(x) = 2 - x$

36. Discuss the applicability o Rolle's Theorem for the function

$f(x) = \log\left[\frac{x^2 + 12}{x}\right]$ *in* $(3,4)$.

37. Verify Langrange's Mean Value theorem for the function

$f(x) = x(x-1)(x-2)$ *in* $\left[0, \frac{1}{2}\right]$

38. Discuss the applicability o Rolle's Theorem for the function

$f(x) = e^x \cos x$ *in* $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

39. Verify Langrange's Mean Value theorem for the function

$f(x) = 2x^2 - 10x + 29$ *in* $[2,7]$.

Q.3 Theory Questions **Marks – 04 / 06**

1. If a function f is continuous on a closed and bound interval [a, b] ,then show that f is bounded on [a, b].

2. Show that every continuous function on closed and bounded interval attains its bounds.

3. Let $f:[a,b]\to R$ be a continuous on [a ,b] and if $f(a)<k<f(b),$ then show that there exists a point $c\in(a,b)$ such that f (x) = k.

4. If f (x) is continuous in [a, b] and $f(a)\neq f(b)$, then show that f assume every value between f (a) and f (b).

5. If a function is differentiable at a point then show that it is continuous at that point. Is converse true? Justify your answer.

6. State and Prove Rolle's theorem OR

   If a function f(x) defined on [a,b] is

   i)continuous on [a,b] ii) Differentiable in (a, b) iii) f (a) = f(b)

   then show that there exists at least one real number $c\in(a,b)$ such that f'(c)=0.

7. State and Prove Langrange's Mean Value Theorem. OR

   If a function f(x) defined on [a,b] is i) continuous on [a,b]

   ii) differentiable in (a, b)

   then show that there exixt at least one real number $c\in(a,b)$ such that

   $$f'(c)=\frac{f(b)-f(a)}{b-a}$$

8. State and Prove Cauchy's Mean Value Theorem. OR

   If f(x) and g(x) are two function defined on [a,b] such that

   i) f(x) and g(x) are continuous on [a, b]

   ii) f(x) and g(x) are differentiable in (a,b)

   iii) $g'(x)\neq 0,\ \forall\ x\in(a,b)$

   then show that there exist at least one real number $c\in(a,b)$ such that

   $$\frac{f'(c)}{g'(c)}=\frac{f(b)-f(a)}{g(b)-g(a)}$$

9. State Rolle's Theorem and write its geometrical interpretation.

10. State Langrange's Mean Value Theorem and write its geometrical interpretation.

11. If f(x) is continuous in [a,b] with M and m as its bounds then show that f(x) assumes every value between M and m.

12. Using Langrange's Mean Value Theorem show that

$$\left|\frac{\cos a\theta - \cos b\theta}{\theta}\right| \leq b - a, if\,\theta \neq 0$$

13. If f(x) be a function uch that $f'(x) = 0, \forall x \in (a,b)$ then show that

f(x) is a constant in this interval.

14. If f(x) is continuous in the interval [a,b] and $f'(x) > 0, \forall x \in (a,b)$ then show that f(x) is monotonic increasing function of x in the interval [a,b].

15. If a function f(x) is such that i) it is continuous in [a, a+h]

ii) it is derivable in (a, a+h)

iii) f(a) = f(a+h)

then show that there exist at least one real number θ such that $f'(a+\theta h) = 0,$

where 0< θ <1.

16. If the function f(x) is such that i) it is continuous in [a, a+h]

ii) it is derivable in (a, a+h)

then show that there exists at least one real number θ such that

$$f(a+h) = f(a) + hf'(a+\theta h), \quad where 0 < \theta < 1$$

17. If f(x) is continuous in the interval [a,b] and $f'(x) < 0, \forall x \in (a,b)$ then show that F(x) is monotonic decreasing function of x in the interval [a, b].

Unit II

Successive Diff. And Taylor's Theorem,

Asymptotes, Curvature and Tracing of Curves

Q-1.Question **(2-marks each)**

1. State Leibnitz theorem for the n^{th} derivative of product of two functions.

2. Write n^{th} derivative of e^{ax}.

3. Write n^{th} derivative of $\sin(ax+b)$.

4. Write n^{th} derivative of $\cos(ax+b)$.

5. State Taylor's theorem with Langrange's form of reminder after n^{th} term.

6. State Maclaurin's infinite series for the expansion of f(x) as power series in [0,x].

7. Define Asymptote of the curve.

8. Define intrinsic equation of a curve.

9. Define point of inflexion.

10. Define multiple point of the curve.

11. Define Double point of the curve.

12. Define Conjugate point of the curve.

13. Define Curvature point of the curve at the point.

Q-2 Examples **(4- marks each)**

1. If $y=\dfrac{x^2+4x+1}{x^3+2x^2-x-2}$, *find* y_n.

2. If $y=e^{ax}\cos^2 x\sin x$, *find* y_n.

3. If $y=x^2\sin(3x+7)$, *find* y_8.

4. If $y=(\sin^{-1}x)^2$ *Prove that*
$(1-x^2)y_{n+2}-(2n+1)y_{n+1}-n^2y_n=0$

5. *If* $y=\cos(m\sin^{-1}x)$ *Prove that*
$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$

6. *If* $y=\tan(\log y)$ *Prove that*
$(1+x^2)y_{n+1}+(2nx-1)y_n+n(n-1)y_{n-1}=0$

7. *If* $y^{1/m}+y^{-1/m}=2x$ *Prove that*
$(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$

8. *If* $\cos^{-1}\left(y/b\right)=\log\left(x/n\right)^n$ *Prove that*
$x^2y_{n+2}+(2n+1)xy_{n+1}+2n^2y_n=0$

9. Find y_n *if* $y=\dfrac{x^2}{(x+2)(2x+3)}$

10. Find y_n *if* $y=\cos^4 x$

11. *If* $y=a\cos(\log x)+b\sin(\log x)$ *Prove that*
$x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$

12. *If* $y=\tan^{-1}x$ *Prove that*
$(1+x^2)y_{n+2}+2(n+1)xy_{n+1}+n(n+1)y_n=0$

13. Find y_n *if* $y = e^x \log x$

14. Find y_n *if* $y = \cos x \cos 2x \cos 3x$

15. *If* $y = \sin^2 x \cos^2 x$ *Prove that*

$$y_n = \frac{-4^n}{8}.\cos\left(4x + n\pi/2\right)$$

16. *If* $y=(x^2-1)^n$ *Prove that*

$$(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$

17. *If* $y = e^{m\cos^{-1}x}$ *Prove that*

$$(x^2-1)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$$

18. *If* $y = \left(x + \sqrt{x^2 - a^2}\right)^m$ *Prove that*

$$(x^2-a^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$$

19. *If* $y = \sin(m\sin^{-1}x)$ *Prove that*

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-m^2)y_n = 0$$

20. *If* $y = \cos(\log x)$ *Prove that*

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

21. Use Taylor's theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of (x-2) .

22. Expand sinx in ascending powers of $\left(x - \pi/2\right)$

23. Assuming the validity of expansion , prove that

$$e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{30} + ----$$

24. Assuming the validity of expansion , prove that

$$\sec x = \frac{x^2}{2!} + \frac{2x^4}{4!} + \frac{16x^5}{6!} + ----$$

25. Expand log(sinx) in ascending powers of (x- 3).

26. Expand tanx in ascending powers of $\left(x - \pi/4\right)$

27. Prove that $\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5$ -------- and hence find the value of π

28. Prove that $\sin^{-1} x = x + 1^2.\frac{x^3}{3!} + 1^2.3^2.\frac{x^5}{5!} + 1^2.3^2.5^2.\frac{x^7}{7!} = ----$

29. Use Taylor's theorem ,Evaluate $\lim_{x\to 0} \frac{2(\tan x - \sin x) - x^3}{x^5}$

30. Expand e^x in ascending powers of (x- 1).

31. Expand $2 + x^2 - 3x^5 + 7x^6$ in power of (x-1).

32. Obtain by Maclurin's theorem the first five term in the expansion of $\log(1 + \sin x)$.

33. Obtain by Maclurin's theorem the expansion of $\log(1 + \sin^2 x)$ *upto* x^4.

34. Assuming the validity of expansion , prove that

$$e^{\sin x} = 1 + x + \frac{1}{2}x^2 - \frac{1}{8}x^4 + ------$$

35. Find the asymptotes of the curve $y = \frac{x}{x^2 - 4}$

36. Find the asymptotes of the curve $y = x - 2 + \frac{x^2}{\sqrt{x^2 + 9}}$

37. Find the asymptotes of the curve $y = 3\sqrt{x^2 - x^3}$

38. Find the asymptotes of the curve $x = t, y = t + 2\tan^{-1} t$

39. Find the asymptotes of the curve $y = \frac{x^2}{x^2 - 4}$

40. Find the asymptotes of the curve $y = \frac{x^3}{x^2 + x - 2}$

41. Find the differential arc and also the cosine and sine of the angle made by the tangent with positive direction of X-axis . for the curve $y^2 = 4ax$.

42. Find the differential of the arc of the curve $r = a\cos^2\left(\theta/2\right)$ .Also find the sine ratio of the angle between the radius vector and the tangent line.

43. Find the point on the parabola $y^2 = 8x$ at which curvature is 0.128.

44. Find the curvature of $r^2 = 2a^2\cos 2\theta, \ \ at \ \ \theta = \pi$.

45. Find the curvature and radius of curvature at a point "t" on the curve, $x = a(\cos t + t\sin t), \quad y = a(\sin t - t\cos t)$.

46. Find the curvature of the curve, $y = x - x^2 \quad at \ P(1,0)$.

47. Find the curvature of the curve, $y = x^4 - 4x^3 - 18x^2 \quad at$ origin .

48. Find the curvature of the curve, $y^3 = x \quad at \ P(1,1)$.

49. Examine for concavity and point of inflection of Guassian Curve y = e^{-x^2}

50. Trace the curve $y = (x-1)^2(x+2)$

51. Trace the curve $y = x(1-x)^3$

52. Find the asymptotes parallel to co-ordinate axes for the curve $y^2(x^2 - a^2) = x$

53. Find the radius of curve of $y = c\tan\psi$.

54. Show that the curvature of the point $\left(3a/2, 3a/2\right)$ on the folium $x^3 + y^3 = 3axy \ \ is \ \ \dfrac{-8\sqrt{2}}{3a}$

55. Find the point on the parabola $y^2 = 8x$ at which radius of curvature is $7\,{}^{13}\!/_{16}$.

56. Examine the nature of the origin of $x^3 + y^3 - 3axy = 0$.

57. Trace the curve $x^3 + y^3 = 3axy$.

58. Trace the curve $xy^2 = a^2(a-x)$.

Unit III

Integration of Irrational Algebraic and Transcendental Functions, Applications of Integration

Q-1 **Marks - 02**

1. The proper substitution for the integral of the type $\int \frac{dx}{(px+q)\sqrt{ax+b}}$ $is-----$

2. Evaluate $\int \frac{dx}{x\sqrt{x^2-4}}$

3. Evaluate $\int \frac{dx}{(1-3x)\sqrt{x+2}}$

4. Evaluate $\int \frac{dx}{(2-x)\sqrt{1-x}}$

5. Evaluate $\int \frac{dx}{x\sqrt{3x+2}}$

6. Evaluate $\int \frac{dx}{(1-2x)\sqrt{2-x}}$

7. Evaluate $\int \frac{dx}{(2x-3)\sqrt{x}}$

8. Evaluate $\int \frac{dx}{(4x+1)\sqrt{x-2}}$

9. Evaluate $\int \frac{\cos x.dx}{(2\sin x-1)\sqrt{2-\sin x}}$

10. Evaluate $\int \frac{e^x dx}{(2e^x+3)\sqrt{e^x-4}}$

11. Reduction formula for $\int_0^{\pi/2} \sin^n x dx$ $is----$

12. Evaluate $\int_0^{\pi/2} \sin^9 x dx$ = -------------

13. Evaluate $\int_0^{\pi/2} \sin^6 x dx$

14. Evaluate $\int_0^{\pi/2} \sin^7 x dx$

15. Reduction formula fpr $\int_0^{\pi/2} \cos^n x dx$ = – – – –

16. Evaluate $\int_0^{\pi/2} \cos^8 x dx$

17. Evaluate $\int_0^{\pi/2} \cos^9 x dx$

18. Evaluate $\int_0^{\pi/4} \sin^4 2x dx$

19. Evaluate $\int_0^{\pi} \sin^5 \frac{x}{2} dx$

20. Evaluate $\int_0^{a} \frac{x^5}{\sqrt{a^2 - x^2}} dx$

21. Evaluate $\int_0^{\infty} \frac{dx}{(a^2 + x^2)^4}$

22. Evaluate $\int_0^{\infty} \frac{dx}{(1 + x^2)^{5/2}}$

23. Evaluate $\int \sin^3 x . \cos^4 x dx$

24. Evaluate $\int \sin^6 x . \cos^5 x dx$

25. Evaluate $\int \sin^4 x . \cos^6 x dx$

26. Evaluate $\int \sin^5 x . \cos^7 x dx$

27. Evaluate $\int_0^{\pi/2} \sin^5 x.\cos^4 x dx$

28. Evaluate $\int_0^{\pi/2} \sin^8 x.\cos^5 x dx$

29. Evaluate $\int_0^{\pi/2} \sin^4 x.\cos^8 x dx$

30. Evaluate $\int_0^{\pi/2} \sin^5 x.\cos^9 x dx$

31. The proper substitution for the integral of the type $\int \frac{dx}{(px^2+qx+r)\sqrt{ax^2+bx+c}}$ is ----------

32. The length of the arc of the curve y = f(x) between the points x = a , x = b is given by S = ---------------- with usual notation.

33. The length s of the arc of the curve x = f (t) ,y = ψ (t) between the points where t = a , t = b is given by S = ------------ with usual notations.

34. The equation of the Catenary is ----------

35. The equation of the Astriod is ----------

36. The volume of the solid generated by revolving about X-axis , the area bounded by the curve y = f(x) , the X- axis and the ordinate x = a , x = b is given by V = ------------- with usual notation .

37. The volume of the solid generated by Revolving about X-axis ,the area bounded by the curve x = g (y) , the Y-axis and the abscissas y = c , y = d is given by V= ---------------with usual notation .

38. The volume of the solid generated by revolving about X-axis , the area bounded by the parametric curve X = ϕ (t) ,Y = ψ (t) and the ordinate t = a , t = b is given by V = ------------- with usual notation .

39. The volume of the solid generated by revolving about Y-axis , the area bounded by the parametric curve Y= ϕ (t) ,Y = ψ (t) and the abscissas t = a , t = b is given by V = ------------- with usual notation.

40. The volume of the solid generated by revolving about X-axis , the area bounded by the curve $Y_1 = \phi$ (x) ,$Y_2 = \psi$ (x) and the ordinates x = a , x = b is given by V = ------------- with usual notation.

41. The Volume of the sphere of radius a is --------------

42. The volume of the ellipsoid formed by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about Y-axis is --------

43. The area of the curved surface of the solid generated by revolving about X-axis , the area bounded by the continuous curve y = f(x) , the X-axis and the ordinates x = a , x = b is S=-------------

44. The area of the curved surface of the solid generated by by revolving about Y-axis , the area bounded by the continuous curve g = f(y) , the Y-axis and the abscissae y = c , y = d is S=-------------

45. The area of the curved surface of the solid generated by by revolving about X-axis , the area bounded by the curve x = ϕ(t), y = ψ (t) , the X-axis and the ordinates t = a , t = b is S =------------- where $\frac{ds}{dt} = -----$

46. The surface area of the sphere of radius a is -----------

47. Write down the parametric equation of the cycloid.

48. $\int_0^1 \frac{x^n}{\sqrt{1-x^2}} dx = \int_0^{\pi/2} ----$

49. $\int_0^\infty \frac{1}{(1+x^2)^n} dx = \int_0^{\pi/2} ----$

50. Define i) A rectification

ii) A cap of the sphere.

Q-2 **(4-marks each)**

Integral of the form $\int \frac{dx}{(px^2+qx+r)\sqrt{ax+b}}$

1. Evaluate $\int \frac{dx}{(x^2+1)\sqrt{x}}$

2. Evaluate $\int \frac{dx}{(x^2-2x+2)\sqrt{x-1}}$

3. Evaluate $\int \frac{dx}{(2x^2-2x+1)\sqrt{2x-1}}$

4. Evaluate $\int \frac{dx}{(x^2+5x+8)\sqrt{x+3}}$

5. Evaluate $\int \frac{dx}{(x^2-2x+2)\sqrt{x-1}}$

6. Evaluate $\int \frac{dx}{(x^2-4x+5)\sqrt{x-2}}$

Integral of the form $\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$

7. Evaluate $\int \frac{dx}{x\sqrt{x^2+x+1}}$

8. Evaluate $\int \frac{dx}{(1-x)\sqrt{x^2+1}}$

9. Evaluate $\int \frac{dx}{x\sqrt{x^2+x+2}}$

10. Evaluate $\int \frac{dx}{(1-x)\sqrt{x^2+2}}$

11. Evaluate $\int \frac{dx}{(1-2x)\sqrt{x^2+x}}$

12. Evaluate $\int \frac{dx}{(x+1)\sqrt{x^2+1}}$

13. Evaluate $\int \frac{dx}{(x-1)\sqrt{x^2+x+1}}$, $(x \geq 1)$

14. Evaluate $\int \frac{dx}{x\sqrt{1-2x-x^2}}$

15. Evaluate $\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$

Integral of the form $\int \frac{dx}{(px^2+qx+r)\sqrt{ax^2+bx+c}}$

16. Evaluate $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

17. Evaluate $\int \frac{dx}{(x^2+4)\sqrt{x^2+1}}$

18. Evaluate $\int \frac{dx}{(x^2-1)\sqrt{x^2+1}}$

19. Evaluate $\int \frac{dx}{(x^2+2)\sqrt{x^2+1}}$

Reduction formula type examples-

20. Evaluate $\int_0^{1/\sqrt{3}} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

21. Evaluate $\int_0^1 x^{9/2}(1-x)^{7/2}\,dx$

22. Evaluate $\int_0^a x^4\sqrt{a^2-x^2}\,dx$

23. Evaluate $\int_0^1 \frac{x^4}{\sqrt{1-x^2}}\,dx$

24. Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^{5/2}}$

25. Evaluate $\int_0^1 \frac{x^8}{\sqrt{1-x^2}}dx$

26. Evaluate $\int_0^1 x^6\sqrt{1-x^2}\,dx$

27. Evaluate $\int_0^1 x^7\sqrt{\frac{1+x^2}{1-x^2}}dx$

28. Evaluate $\int_0^4 x\sqrt{4x-x^2}\,dx$

29. Evaluate $\int_0^\infty \frac{x^3}{(1+x^2)^{7/2}}dx$

30. Evaluate $\int_0^\infty \frac{x^2}{(1+x^2)^{7/2}}dx$

31. Evaluate $\int_0^\infty \frac{x^7}{(1+x^2)^{3/2}}dx$

32. Evaluate $\int_0^\infty \frac{x^5}{(1+x^2)^{9/2}}dx$

33. Evaluate $\int_0^\infty \frac{x^4}{(1+x^2)^4}dx$

34. Evaluate $\int_0^\infty \frac{x^3}{(1+x^2)^3}dx$

35. Evaluate $\int_0^\infty \left(\frac{x}{1+x^2}\right)^6 dx$

36. Evaluate $\int_0^\infty \left(\frac{x}{1+x^2}\right)^5 dx$

37. Show that $\int_0^4 x^2\sqrt{4x-x^2}\,dx = 10\pi$

38. show that $\int_0^1 x^2\sqrt{x-x^2}\,dx = \frac{5\pi}{128}$

39. show that $\int_0^6 x\sqrt{6x-x^2}\,dx = \frac{27\pi}{2}$

40. show that $\int_0^2 x^3\sqrt{2x-x^2}\,dx = \frac{7\pi}{8}$

41. Let $I_n = \int \frac{\sin nx}{\sin x} dx, n > 1$ show that

$I_n = \frac{2\sin(n-1)x}{n-1} + I_{n-1}$ Where n is a positive integer.

42. $Show\ that \int \frac{\sin 6x}{\sin x} dx = 2\left[\frac{\sin 5x}{5} + \frac{\sin 3x}{3} + \sin x\right]$

$Hence\ Show\ that \int_0^{\pi} \frac{\sin 6x}{\sin x} dx = 0$

43. $Show\ that \int \frac{\sin 7x}{\sin x} dx = 2\left[\frac{\sin 6x}{6} + \frac{\sin 4x}{4} + \frac{\sin 2x}{2} + \frac{x}{2}\right]$

$Hence\ Show\ that \int_0^{\pi} \frac{\sin 7x}{\sin x} dx = \pi$

44. $Let\ \ I_{22} = \int \frac{\sin 22x}{\sin x} dx,$

$Show\ \ that\ \ I_{22} = 2\left(\frac{\sin 21x}{21} + \frac{\sin 19x}{19}\right) + I_{18}$

45. $Show\ that \int \frac{\sin 5x}{\sin x} dx = \sin 2x(3 - 2\sin^2 x) + x$

Q3. **(6-marks each)**

Reduction formulae

1. Evaluate $\int \sin^m x.\cos^n x dx$, where m, n are positive integers.

2. Evaluate $\int_0^{\pi/2} \sin^n x dx$, where n is positive integers.

3. Evaluate $\int_0^{\pi/2} \cos^n x dx$, where n is positive integers.

4. Evaluate $\int_0^{\pi/2} (\sin x)^m.(\cos x)^n dx$, where m and n are positive integers.

5. Evaluate $\int_0^{\infty} \frac{1}{(1+x^2)^{n+1/2}} dx$, where n is a positive integers.

Application of Integration.

Rectification –

6. Show that the length of an arc of the parabola $y^2 = 4ax$ cutoff by the y = 2x is $\left[\sqrt{2} + \log(1+\sqrt{2})\right]$.

7. Show that the length of an arc of the parabola $x^2 = y$ form the vertex to any extremity of the latus rectum is $\frac{1}{2\sqrt{2}} + \frac{1}{4}\log(1+\sqrt{2})$.

8. Show that the length of the arc of the curve $y = x^2$ cutoff by the line x – y = 0 is $\frac{1}{4}\left[2\sqrt{5} + \log(2+\sqrt{5})\right]$.

9. Find the length of an arc of the catenary $y = \frac{c}{2}\left(e^{x/c} + e^{-x/c}\right)$ measured from the vertex (0 , c) to any point (x, y).

10. Find the length of an arc of the curve $y = \sin^{-1} e^x$ between the points where $y = \frac{\pi}{6}$ *and* $y = \frac{\pi}{2}$.

11. Using theory of integration , obtains the circumference of the circle $x^2 + y^2 = 25$.

12. Find the length of an arc of the cycloid $x = a(\theta - \sin\theta), y = e^{\theta}\left(\cos\frac{\theta}{2} - 2\sin\frac{\theta}{2}\right)$ between the cups $\theta = 0$ *and* $\theta = 2\pi$.

13. Find the length of an arc of the curve $x = e^{\theta}\left(\sin\frac{\theta}{2} + 2\cos\frac{\theta}{2}\right)$, $y = e^{\theta}\left(\cos\frac{\theta}{2} - 2\sin\frac{\theta}{2}\right)$ between the cups $\theta = 0$ *and* $\theta = \pi$.

14. Find the length of an arc of the curve $x = a(2\cos\theta - \cos 2\theta)$, $y = a(2\sin\theta - six2\theta)$, measured from the points, where $\theta = 0$ *and* $\theta = \pi$ is 89.

15. Find the length of an arc of the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, from the points, where $\theta = 0$ *and* $\theta = 2\pi$ *is* $2\pi^2 a$.

Volumes of Solids of Revolution

16. Using theory of integration , show that the volume of sphere of radius 'a ' is $\frac{4}{3}\pi a^3 \textit{cubic units}$.

17. Show that the volume of solid genered by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, about X-axis is $\frac{4}{3}\pi ab^2 \textit{cubic units}$.

18. Find the Volume of the solid formed by revolving the arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about its base.

19. The area enclosed by the hyperbola xy = 12 and the line x + y =7 is revolved about X-axis , Show that the volume of the solid generated is $\frac{\pi}{3} \textit{cubic units}$

20. Compute the volume of the solid generated by revolving about Y-axis , the region enclosed by the parabolas $y = x^2$ *and* $8x = y^2$.

Areas of surface s of revolution-

21. The are of the parabola $y^2 = x$ between the origin and the point (1,1) is revolved about X-axis , Find the area of the surface of revolution of the solid formed .

22. Find the surface area of the solid generated by the revolution about the X-axis of the loop of the curve $x = t^2, y = t - \frac{t^3}{3}$.

23. The arc of the parabola $y^2 = 4x$ between its vertex and an extremity of its latus rectum revolves about its axis. Find the surface area traced out.

24. If the segment of a straight line y = 2x between x = 0 to x = 1 is revolved about Y-axis .show that surface area of the solid so formed is $4\sqrt{5}\pi$ *square units* .

25. Find the area of the surface generated when the segment of the straight line y = x between x = 0 to x = 1 is revolved about Y-axis.

Unit – IV

Differential Equation of First Order & First Degree

Q-1 04 or 06 Marks

1. Explain the method of solving homogeneous diff. Equation of the type $Mdx + Ndy = 0$, where $M = M(x, y)$, $N = N(x, y)$
2. Explain the method of solving non-homogeneous diff. Equation $\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$, where $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers.
3. Explain the method of solving exact diff. Equation $Mdx + Ndy = 0$, where $M = M(x, y)$, $N = N(x, y)$
4. If the diff. Eq. $Mdx + Ndy = 0$ is homogeneous then $\frac{1}{Mx+Ny} = 0$ is an integrating factor, where $Mx + Ny \neq 0$ and $M = M(x, y)$, $N = N(x, y)$
5. If the diff. Eq. $Mdx + Ndy = 0$ is of type $f_1(x, y)ydx + f_2(x, y)xdy = 0$ then $\frac{1}{Mx-Ny} = 0$ is an integrating factor, where $Mx - Ny \neq 0$.
6. IF $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$ is a function of x alone then $e^{\int f(x)dx}$ is an integrating factor of equation $Mdx + Ndy = 0$ where $M = M(x, y)$, $N = N(x, y)$
7. IF $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$ is a function of y alone then $e^{\int f(y)dy}$ is an integrating factor of equation $Mdx + Ndy = 0$ where $M = M(x, y)$, $N = N(x, y)$
8. Solve the linear diff. Equation $\frac{dy}{dx} + Py = Q$, where P & Q are functions of x only.
9. Solve the linear diff. Equation $\frac{dx}{dy} + Px = Q$, where P & Q are functions of y only.
10. Explain the method of solving the diff. Equation $F(x, y, p) = 0$, which is solvable for p, where $p = \frac{dy}{dx}$.
11. Explain the method of solving the diff. Equation $F(x, y, p) = 0$, which is solvable for y, where $p = \frac{dy}{dx}$.
12. Explain the method of solving the diff. Equation $F(x, y, p) = 0$, which is solvable for x, where $p = \frac{dy}{dx}$.

Q-2 04 Marks

Solve the following differentials equations

1. $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$
2. $y \sec^2 x \, dx + (y+7) \tan x \, dy = 0$
3. $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y} + y\cos y$
4. $(y - x\frac{dy}{dx}) = a(y^2 + \frac{dy}{dx})$
5. $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

Solve the homogeneous diff. Eq.

6. $(x^3+y^3)dx - 3xy^2dy = 0$
7. $x^2dy + (y^2-xy)dx = 0$
8. $(x^2+xy-y^2)\,dy + (2xy - 3y^2)dx = 0$
9. $xdy - ydx = \sqrt{x^2 + y^2}dx$
10. $x^2 \frac{dy}{dx} = y(x+y)/2$
11. $(x^2-y^2)\,dx + 2xy\,dy = 0$
12. $\frac{dy}{dx} = \frac{(x^2 - xy + y^2)}{xy}$
13. $\frac{dy}{dx} = \frac{(x^2 - y^2)}{2xy}$
14. $(x^2+y^2)\frac{dy}{dx} = xy$
15. $(x + y \cot x/y)\, dy - y\, dx = 0$

Solve the Non-homogeneous diff. Eq.

16. $\frac{dy}{dx} = \frac{(2x - 5y + 3)}{(2x+4y-6)}$
17. $(2x - y + 1)\, dx + (2y - x - 1)dy = 0$
18. $\frac{dy}{dx} = \frac{(6x - 4y + 3)}{(3x-2y+1)}$
19. $\frac{dy}{dx} = \frac{(x+ y+1)}{(x+y-1)}$
20. $\frac{dy}{dx} = \frac{(x + 2y + 1)}{(2x+4y-6)}$

21. $\frac{dy}{dx} = \frac{(x+2y+3)}{(2x+3y+4)}$

22. $\frac{dy}{dx} = \frac{(y-x+1)}{(y+x+5)}$

23. $\frac{dy}{dx} = \frac{(2x-y+1)}{(x+2y-3)}$

24. $\frac{dy}{dx} = \frac{(4x-6y+3)}{(6x-9y-1)}$

Solve the exact diff. Eq.

25. $(2x^2 + 3y)dx + (3x + y - 1)\, dy = 0$

26. $\frac{dy}{dx} + \frac{y\cos x+\sin y+y}{\sin x+x\cos y+x} = 0$

27. $(x^2 + y^2 - a^2)\, x\, dx + (x^2 - y^2 - b^2)\, y\, dy = 0$

28. $(1 + e^{\frac{x}{y}})\, dx + [e^{\frac{x}{y}}\, (1 - x/y]\, dy = 0$

29. $(\sec x \tan x \tan y - e^x)\, dx + \sec x \sec^2 x\, dy = 0$

30. $(x^2 - 4xy - 2y^2)\, dx + (y^2 - 4xy + 2x^2)\, dy = 0$

31. $(e^y + 1) \cos x\, dx + e^y \sin x\, dy = 0$

32. $(\sin x \cos y + e^{2x})\, dx + (\cos x \sin y + \tan y)\, dy = 0$

33. $[x\sqrt{x^2+y^2} - y]\, dx + [y\sqrt{x^2+y^2} - x]\, dy = 0$

34. $[\cos x \tan y + \cos(x + y)]\, dx + [\sin x \sec^2 y + \cos(x + y)]\, dy = 0$

Solve the Non-exact diff. Eq.

35. $(x^2y - 2xy^2)\, dx - (x^3 - 3x^2y)\, dy = 0$

36. $(x^2 - 5xy + 7y^2)\, dx + (5x^2 - 7xy)\, dy = 0$

37. $(x^2y^2 + 4xy + 2)\, x\, dx - (x^2y^2 + 5xy + 2)\, ydy = 0$

38. $(3xy^2 - y^3)\, dx - (2x^2y - xy^2)\, dy = 0$

39. $(1 + xy)\, ydx + (1 - xy)\, xdy = 0$

40. $(xy \sin xy + \cos xy)\, ydx + (xy \sin xy - \cos xy)\, xdy = 0$

41. $y(xy + 1)\, dx + x(1 + xy + x^2y^2)\, dy = 0$

42. $(xy + 2x^2y^2)\, ydx + (xy - x^2y^2)\, xdy = 0$

43. $(1/x+y)\, dx + (1/y-x)\, dy = 0$

44. $(x^4y^4 + x^2y^2 + xy)\, ydx + (x^4y^4 - x^2y^2 + xy)\, xdy = 0$

45. $(x^2 + y^2)\, dx - 2xy\, dy = 0$

46. $(x^2y^2 + 2xy + 1)\, ydx + (x^2y^2 - xy + 1)\, xdy = 0$

47. $(1 + xy)\ ydx + (1 - xy)\ xdy = 0$

48. $(xy^3 + y)\ dx + 2(x^2y^2 + x + y^4)\ dy = 0$

49. $(y^4 + 2y)\ dx + (xy^3 + 2y^4 - 4x)\ dy = 0$

50. $(x - y^2)\ dx + 2xy\ dy = 0$

51. $(3x^2y^4 + 2xy)\ dx + (2x^2y^3 - x^2)\ dy = 0$

52. $(x^2y + y^3)\ dx + (2/3\ x^3 + 4xy^2)\ dy = 0$

53. $(x^4e^x - 2mxy^2)\ dx + 2mx^2y\ dy = 0$

54. $(x^2 + y^2 + x)\ dx + xy\ dy = 0$

55. $(x^2 + y^2 + 2x)\ dx + 2y\ dy = 0$

56. $(x - y^2)\ dx + 2xy\ dy = 0$

57. $(x^3 + xy^4)\ dx + 2y^3\ dy = 0$

58. $(2y^2 + 3xy - 2y + 6x)\ dx + x(x + 2y - 1)\ dy = 0$

59. $2y\ (x + y + 2)\ dx + (y^2 - x^2 - 4x - 1)\ dy = 0$

60. $(7x^4y + y + 2)\ dx + (x^4 + xy)\ x\ dy = 0$

Solve the Linear diff. Eq.

61. $\frac{dy}{dx} - 2y = e^{2x}$

62. $\frac{dy}{dx} + x^2y = x^5$

63. $\sin x \frac{dy}{dx} + 3y = \cot x$

64. $\frac{dy}{dx} + 2xy + xy^4 = 0$

65. $3y^2\ \frac{dy}{dx} + 2xy^3 = 4xe^{-x^2}$

66. $(x^2y^3 - xy)\ dy = dx$

67. $xy - \frac{dy}{dx} = y^3\ e^{-x^2}$

68. $\frac{dy}{dx} = x(x^2 - 2y)$

69. $\frac{dy}{dx} = (2x + 3y - 7)^2$

70. $\cos x \frac{dy}{dx} + 2y \sin x = \sin x \cos x$

Solve the following diff. Eq. for x, y, p

71. $p^2 - 5p + 6 = 0$
72. $p - 1/p = x/y - y/x$
73. $p(p + y) = x(x+ y)$
74. $p(p - y) = x(x+ y)$
75. $p^2 - 7p + 12 = 0$
76. $2y = ax/p + px$
77. $4y = x^2 + p^2$
78. $3x - y + \log p = 0$
79. $y = 2px + x^2p^4$
80. $y - 2px = f(xp^2)$
81. $y = 2px + p^2y$
82. $p^3 - 2xyp + 4y^2 = 0$
83. $y = 3px + 6y^2p^2$
84. $y = 2px + y^2p^3$
85. $xyp^2 + (x^2 + xy + y^2)p + x(x + y) = 0$
86. $3x - y + \log p = 0$
87. $y = (1 + p)x + p^2$
88. $y^2 \log y = xyp + p^2$
89. $xp^3 = m + np$

Que. 3 02 Marks

Write the definition of following

1. Homogeneous differential equation
2. Non- homogeneous differential equation
3. Exact differential equation
4. Linear differential equation
5. Bernaoll's differential equation
6. Claraut's differential equation

Find the integrating factor of the following differential equation

7. $(1 + y^2)\, dx + (x - e^{tan^{-1}y})\, dy = 0$
8. $\frac{dy}{dx} + \frac{4x}{x^2+1} y = \frac{1}{(x^2+1)^3}$
9. $\frac{dy}{dx} - \frac{1}{1+x} \tan y = (1 + x)\, e^x \sec y$
10. $(x \cos x) \frac{dy}{dx} + (x \sin x + \cos x)\, y = 1$
11. $\frac{dy}{dx} = x^3y^3 - xy$
12. $(xy^3 + y)\, dx + 2(x^2y^2 + x + y^4)dy = 0$
13. $(x^2 + y^2 + 2x)\, dx + 2ydy = 0$
14. $(y^4 + 2y)\, dx + (xy^3 + 2y^4 – 4x)\, dy = 0$

Multiplying by appropriate integrating factor, make following diff. Eq. Exact.

15. $(x^2y^2 + 2)\, ydx + (2 – 2x^2y^2)\, dy = 0$
16. $(x^2y^2 + xy + 1)\, ydx + (x^2y^2 – xy + 1)\, dy = 0$
17. $(3xy^2 – y^3)\, dx - (2x^2y - xy^2)\, dy = 0$
18. $(x^2 + y^2)\, dx – 2xy\, dy = 0$
19. $(7x^4y + 2xy^2 – x^3)\, dx + (x^4 + xy)\, xdy = 0$
20. $(x^2 + y^2 + x)\, dx + xy\, dy = 0$

Write the appropriate answer of the following, where P & Q are functions of x only.

21. The diff. Eq. $\frac{dy}{dx}$ + Py dy = Q is ---

A) Linear D. E. B) Bernaoll's D. E.

C) Exact D. E. D) Not exact D. E.

22. The diff. Eq. $(x^2 + y^2)\frac{dy}{dx} = xy$ is ---

A) Linear D. E. B) Homogeneous D. E.

C) Bernaoll's D. E. D) Non- homogeneous D. E.

22. The diff. Eq. (1 + xy) ydx + (1 - xy) xdy = 0

A) Not exact D. E. B) Clairaut's D. E.

C) Linear D. E. D) Non- homogeneous D. E.

24. The diff. Eq. $3\frac{dy}{dx} + \frac{2}{x+1}y = \frac{x^3}{y^3}$ is ---

A) Not exact D. E. B) Clairaut's D. E.

C) Linear D. E. D) Homogeneous D. E.

25. The diff. Eq. $y = px + \sqrt{4 + p^2}$ is ---

A) Non-homogeneous D. E. B) Clairaut's D. E.

C) Bernaoll's D. E. D) Homogeneous D. E.

Unit V

Differential Equations

Q-1. Questions **2 - Marks**

1. Let f(D)y = X be the L.D.E. If x = 0 with constant coefficient. Then

i) f (D)y = 0 is called ---

ii) f (D) = 0 is called ---

2. If $m_1, m_2, -----m_n$ are n distinct real roots of A.E. f(D) = 0 then G.S. of the equation f(D)y = 0 is ---

3. If $m_1 = m_2$ two root of f(D) = 0, then C. F. of f(D)y = 0 is ---

4. If $m_1 = \alpha + i\beta$ *and* $m_2 = \alpha - i\beta$ are the complex roots of the f(D) = 0 , then G.S. of f(D)y = 0 is ---

5. If $f(D) = (D - m_1)(D - m_2) - - - - - (D - m_n)$, then the G.S. of the L.D.E. f(D)y = 0 is --- .

6. If $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{2x}$,then what is its complementary function ?

7. If $f(D^2)$ is polynomial in D^2 with constant coefficients and

$F(-a)^2 \neq 0$ then i) $\frac{1}{f(D^2)}\cos(ax+b) = ?$

ii) $\frac{1}{f(D^2)}\sin(ax+b) = ?$

8. If $D = \frac{d}{dx}$ and f(D) is a polynomial in D with constant coefficients then

$i)\frac{1}{f(D)}e^{ax} \times V = ?$

$ii)\frac{1}{f(D)} \times V = ?$

where V is function of x .

9.

$i)\ \frac{1}{(D^2+a^2)^r}\cos(ax)=?$

$ii)\ \frac{1}{(D^2+a^2)}\sin(ax)=?$

10. Let $(D^2+4)y=\cos 2x$, find P.I.

11. If $D=\frac{d}{dx}$ and f(D) is a polynomial in D with constant coefficients then .

$\frac{1}{f(D)}e^{ax}=?, f(a)\neq 0$

12.

$i)\ \frac{1}{(D-a)^r}e^{ax}=?$

$ii) If\ \ f(D)=(D-a)^r\phi(D)\ \ and\ \ \phi(a)\neq 0, then \frac{1}{f(D)}e^{ax}=?$

Q-2. Define the following

1. Linear differential equation with constant co-efficients of order n.

2. Associated D.E. and Auxillary equation.

3. Inverse Operator

4. Homogeneous Linear Differential equation of the order n.

Q-3. Multiple choices

1. If $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^{2x}$ is a linear differential equation ,then C.F. is ----

$a)(c_1 + c_2 x)e^x$
$b)(c_1 x + c_2 x^2)e^x$
$c)(c_1 + c_2)e^x$
$d)none\ of\ these$

2. If $(D^3 + 3D^2 + 3D + 1)y = e^{-x}$ is a linear differential equation then C.F. is

$a)(c_1 x + c_2 x + c_3 x^2)e^{-x}$
$b)(c_1 + c_2 x + c_3 x^2)e^{-x}$
$c)(c_1 + c_2 + c_3 x)e^{-x}$
$d)none\ of\ these$

3. If $(D^2 + 2D + 3)y = x - 2x^2$ is a linear differential equation then C.F. is ---

$a)e^{-x}(c_1 \cos\sqrt{2x} + c_2 \sin\sqrt{2x})$
$b)e^{-x} + (c_1 \cos\sqrt{2x} + c_2 \sin\sqrt{2x})$
$c)e^{-x}(c_1 \cos\sqrt{2x} + ic_2 \sin\sqrt{2x})$
$d)none\ of\ these$

4. If $(D^2 + 4)y = \cos 2x$ is a linear differential equation then C.F. is ------

$a)c_1 \cos 2x + c_2 \sin 2x$
$b)c_1 \cos 2x + ic_2 \sin 2x$
$c)c_1 \sin 2x + ic_2 \cos 2x$
$d)none\ of\ these$

5. If $(D^2 + 2) = \cos 2x$ is a linear differential equation then P.I. is ------

$a)\frac{x \sin\sqrt{2x}}{2\sqrt{2}}$
$b)\frac{\sin\sqrt{2x}}{2}$
$c)\frac{x \sin\sqrt{2x}}{\sqrt{2}}$
$d)\frac{x \cos 2x}{2}$

6. If $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0$ is a homogeneous L.D.E.,then solution of L.D.E. is ------

$a)\, y=c_1e^{2z}+c_2e^{-2z}$
$b)\, y=c_1e^{4z}+c_2e^{-4z}$
$c)\, y=c_1e^{2z}+c_2e^{2z}$
$d)$ *none of these*

7. If $(D^2+4)^2y=\cos^2 x$ is a linear differential equation then C.F. is -------

$a)(c_1+c_2)\cos 2x+(c_3x+c_4x)\sin 2x$
$b)(c_1+c_2x)\cos 2x+(c_3+c_4x)\sin 2x$
$c)(c_1x+c_2x^2)\cos 2x+(c_3x+c_4x^2)\sin 2x$
$d)$ *none of these*

8. If $\frac{d^2y}{dx^2}+4y=0$ is a linear differential equation then G.S. is -------

$a)\, A\cos 2x+B\sin 4x$
$b)\, A\cos 2x+B\sin 2x$
$c)\, A\sin 2x+B\cos 4x$
$d)$ *none of these*

9. If $(D^2-6D+13)y=0$ is a linear differential equation then G.S. is -------

$a)\, e^{3x}(A\cos 2x+B\sin 2x)$
$b)\, e^{3x}(A\cos 4x+B\sin 4x)$
$c)\, e^{3x}(\cos 2x+B\sin 2x)$
$d)$ *none of these*

10. If $x^2\frac{d^2y}{dx^2}$ - 3x $\frac{dy}{dx}$ + 4y = 0 is a homogeneous L.D.E. , then G.S. is ------

i) $(c_1 + c_2 \log x)\, x^2$

ii) $x^3\, e^{3x}$

iii) $x\, e^{3z}$

iv) z^2 ez

Q-4. Numerical Examples **04 Marks**

1) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

2) Solve $\frac{d^3y}{dx^3} - 13\frac{dy}{dx} + 12y = 0$

3) Solve $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

4) Solve $2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 12y = 0$

5) Solve $\frac{d^4y}{dx^4} + 4y = 0$

6) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{2x}$

7) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$

8) Solve $\frac{d^2y}{dx^2} + y = 0$

9) Solve $(D^3 - 6D^2 + 9D)y = 0$

10) Solve $(D^4 + 8D^2 + 16)y = 0$

11) Solve $(D-1)^2(D^2+1)y = 0$

12) Solve $(D^2+4)y = \cos 2x$

13) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{2x}$

14) Solve $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cosh 2x$

15) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$

16) Solve $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 4e^{-x/2}$

17) Solve $\frac{d^2y}{dx^2} - 9y = e^{2x} + x^2$

18) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x$

19) Solve $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cosh x$

20) Solve $\frac{d^3y}{dx^3} - y = (1+e^x)^2$

21) Solve $\left[\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y\right] = e^{-2x} + x^2$

22) Solve $\frac{d^3y}{dx^3} + 8y = x^4 + 2x + 1$

23) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = x^2$

24) Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 2x^3 - 3x^2 + 1$

25) Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} - 8y = e^{-2x} + x^2$

26) Solve $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16 = \cos^2 x$

27) Solve $\frac{d^4y}{dx^4} - a^4y = \cos ax$

28) Solve $\frac{d^4y}{dx^4} + y = \sin x \sin 2x$

29) Solve $\frac{d^3y}{dx^3} + y = \cos 2x$

30) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin e^x$

31) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\sin x$

32) Solve $(D^2 - 5D + 6)y = e^{3x}$

33) Solve $(D^2 + 13D + 36)y = e^{-4x} + \sinh x$

34) Solve $(D^3+3D^2+3D+1)y=e^{-x}$

35) Solve $(D^3-5D^2+8D-4)y=e^{2x}$

36) Solve $(D^2-2D+1)y=e^x$

37) Solve $(D^2-4D+4)y=\sinh 2x$

38) Solve $(D^3-4D)y=2\cosh 2x$

39) Solve $(D^3-5D^2+8D-4)y=e^{2x}+3e^x$

40) Solve $(D^3+3D^2+2D)y=x^2$

41) Solve $(D^2+2D+3)y=x-2x^2$

42) Solve $(D^2-D-2)y=1-2x-9e^{-x}$

43) Solve $(D^3+3D^2+2D)y=x^2+4x+8$

44) Solve $(D^2-4D+4)y=8(x^2+e^{2x})$

45) Solve $(D^2-3D+2)y=2x^2-9x^2+6x$

46) Solve $(D^2-4D+3)y=2\cos x+4\sin x$

47) Solve $(D^3+D^2-D-1)y=\sin x$

48) Solve $(D^3+D)y=\sin 3x$

49) Solve $(D^2+4)y=\cos 2x$

50) Solve $(D^4-1)y=\cos x\cos 3x$

51) Solve $(D^2+4)y=\sin 3x+e^x+x^2$

52) Solve $(D^3+D)y=\cos x$

53) Solve $(D^2-1)y=10\sin^2 x$

54) Solve $(D^2+1)y=12\cos^2 x$

55) Solve $(D^3-D^2-6D)y=\cos x+x^2$

56) Solve $(D^3-D^2-D+1)y=\cosh x+\sin x$

57) Solve $(D^2-2D+2)y=x^2e^{3x}$

58) Solve $(D^2-4D+3)y=e^x\cos 2x$

59) Solve $(D^2-6D+13)y=e^{3x}\sin 2x$

60) Solve $(D^2-2D+4)y=e^x\cos^2 x$

61) Solve $(D^2-2D+1)y=\frac{4}{x^2}e^x$

62) Solve $(D^2-1)y=x^2\cos x$

63) Solve $(D^2-1)y=x^2\sin x$

64) Solve $(D^4-1)y=\cos x\cosh x$

65) Solve $(D^4-1)y=e^x\cos x$

66) Solve $(D^3+1)y=e^{2x}\sin x+e^{x/2}\sin\left(\frac{x\sqrt{3}}{2}\right)$

67) Solve $(D^3-7D-6)y=e^{2x}(1+x^2)$

68) Solve $(D^4-2D^3-3D^2+4D+4)y=e^x x^2$

69) Solve $(D^2-1)y=x\sinh x$

70) Solve $(D^2-1)y=xe^{2x}$

71) Solve $(D^2+1)y=x\cos^2 x$

72) Solve $(D^2+4)y=x\sin x$

73) Solve $(D^2-1)y=x^2\cos x$

74) Solve $(D^2+1)y=x\cos 2x$

75) Solve $(D^2+2D+2)y=x\cos x$

76) Solve $(D^2+3D+2)y=x\sin 2x$

77) Solve $(D^2+D)y=(1+e^x)^{-1}$

78) Solve $(D^2+5D+6)=e^{-2x}\sec^2 x(1+2\tan x)$

79) Solve $(D^2-2D+1)=xe^x\sin x$

80) Solve $(D^2-9D+18)y=e^{e^{-3x}}$

81) Solve ($D^2 + 3D + 2$)y = sin e^x

82) Solve ($D^2 + 3D + 2$)y = e^{e^x}

83) Solve ($D^2 + 4$)y = tan 2x

84) Solve ($D^2 + 3D + 2$)y = sin e^{-x}

85) Solve ($D^2 - 2D + 2$)y = x e^x cosx

86) Solve ($D^2 - 1$)y = $(1 + e^{-x})^{-2}$

87) Solve ($x^2D^2 + xD - 4$)y = 0

88) Solve ($x^2D^2 - 3xD + 4$)y = $2x^2$

89) Solve ($D^2 – 1/x\, D + 1/x^2$)y = $(2/x^2)$ Logx

90) Solve ($x^2D^2 - xD - 3$)y = x^2 Logx

91) Solve ($x^2D^2 - 3xD + 5$)y = x^2 sin (Logx)

92) Solve [$(2x+1)^2 D^2 – 2(2x+1)D - 12$]y = 6x

93) Solve [$(1+x)^2D^2 + (1+x)D + 1$]y = 4 cos [Log (2 + x)]

94) Solve ($x^2D^2 + 4xD + 2$)y = e^x

95) Solve [$(2x-1)^3 D^3 + (2x-1)D - 2$]y = 0

96) Solve [$(3x+2)^2 D^2 + 3(3x+2)D - 36$]y = $3x^2 + 4x + 1$

97) Solve [$(1+x)^2 D^2 + (1+x)D + 1$]y = 2 sin[log(1+x)]

98) Solve [$(x+3)^2 D^2 – 4(x+3)D + 6$]y = log (x + 3)

99) Solve [$(x+2)^2 D^2 – (x+2)D + 1$]y = 3x + 4

Q-5. Theory Questions **06 Marks**

1) If $D=\frac{d}{dx}$ and f(D) is a polynomial in D with constant coefficients ,then

Prove that $\frac{1}{f(D)}\, e^{ax} = \frac{1}{f(a)}\, e^{ax},\ if\ f(a) \neq 0$

2) Prove that

$$\frac{1}{(D-a)^r}e^{ax}=\frac{x^r e^{ax}}{r!}$$

$$Hence\frac{1}{f(D)}e^{ax}=\frac{x^r e^{ax}}{r!\phi(a)},\quad if\ f(D)=(D-a)^r\phi(D)\ \&\ \phi(a)\neq 0$$

3) If $f(D^2)$ is polynomial in D^2 with constant coefficients and $f(-a^2)\neq 0$

then prove that $\frac{1}{f(D^2)}\cos(ax+b)=\frac{\cos(ax+b)}{f(-a^2)}$

4) If $f(D^2)$ is polynomial in D^2 with constant coefficients and $f(-a^2)\neq 0$

then $\frac{1}{f(D^2)}\sin(ax+b)=\frac{\sin(ax+b)}{f(-a^2)}$

5) Prove that $\frac{1}{(D^2+a^2)^r}\cos ax=\frac{(-1)^r x^r}{r!(2a)^r}\cos\left(ax+\frac{r\pi}{2}\right), r\in N$

6) Prove that $\frac{1}{(D^2+a^2)^r}\sin ax=\frac{(-1)^r x^r}{r!(2a)^r}\sin\left(ax+\frac{r\pi}{2}\right), r\in N$

7) If $D=\frac{d}{dx}$ and f(D) is a polynomial in D with constant coefficients ,then

Prove that $\frac{1}{f(D)}e^{ax}V=e^{ax}\frac{1}{f(D+a)}V,$ where V is a function of x.

8) If $D=\frac{d}{dx}$ and f(D) is a polynomial in D with constant coefficients ,then

Prove that $\frac{1}{f(D)}xV=\left[x-\frac{1}{f(D)}f'(D)\right]\frac{1}{f(D)}V,$ where Vis a function of x.

9) Define a homogeneous linear differential equations & explains the methods solving it.