# North Maharashtra University, Jalgaon
# Question Bank

## (New syllabus w.e.f. June 2007)

## Class: F. Y. B. Sc.

## Subject: Mathematics

***Paper III(A) ( Vector Analysis & Geometry)***

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# Unit I

## (Product of vectors, Vector Function)

**Q.1 Objective questions** **(2 marks each)**

***A) Fill in the blanks***

i. Vector triple product is a - - - - - - - quantity.
ii. Scalar product of four vectors is - - - - - - - quantity.
iii. Vector product of four vectors is - - - - - -- - quantity.
iv. Every differentiable vector function is continuous is true or false - - - - - - -.
v. Every continuous function is differentiable is true or false - - - - - - -.
vi. $If\ \bar{A}.\bar{B}\times\bar{C}=0\ then\ \bar{A},\ \bar{B}\ \&\ \bar{C}\ are - - - -$.
vii. $(Magnitude\ of\ acceleration)^2 = (- - - - -)^2 + (- - - - -)^2$
viii. $\bar{\imath}\times\bar{\imath}=\bar{\jmath}\times\bar{\jmath}=\bar{k}\times\bar{k}= - - -$ .
ix. $\bar{\imath}\times\bar{\jmath}= - - -,\ \bar{\jmath}\times\bar{k}= - - -,\ \bar{k}\times\bar{\imath}= - - -$.
x. $If\ \bar{u}(t) is\ constant\ vector\ function\ then\ \frac{d\bar{u}}{dt} = - - - -$.

***B) Define***

i. Scalar triple product.
ii. Vector triple product.
iii. Scalar product of four vectors
iv. Vector product of four vectors
v. Reciprocal system of three vectors.
vi. Vector function of one variable.
vii. Continuity of vector function of one scalar variable
viii. Continuity of vector function of two scalar variable
ix. Derivative of vector function.
x. Partial derivative of vector function.

***C) Multiple choice questions***

i. $[\bar{\imath}\ \ \bar{\jmath}\ \ \bar{k}] = - - - -$
   a) 0 b) 1 c) 2 d) 3
ii. $If\ \bar{a}=\bar{\imath},\ \bar{b}=\bar{\jmath},\ \bar{c}=\bar{k}\ then\ \bar{a}\times(\bar{b}\times\bar{c}) = - - -$
   a) 0 b) 1 c) 2 d) 3
iii. $\bar{a}.\bar{a}' = - - - -$

a) 0 b) 1 c) $\bar{a}^2$ d) $|\bar{a}|^2$

iv. $\bar{a}.\bar{b}' = - - -$

a) 0 b) 1 c) $a^2$ d) $b^2$

v. $\bar{\imath}' = - - - -$

a) 0 b) 1 c) $\bar{\imath}$ d) $\bar{\jmath}$

vi. $\bar{a}.\bar{a}' + \bar{b}.\bar{b}' + \bar{c}.\bar{c}' = - - - -$

a) 0 b) 1 c) 3 d) $a^2 + b^2 + c^2$

vii. $If\ \bar{u}.\frac{d\bar{u}}{dt} = 0\ then\ the\ vector\ function\ \bar{u}(t) is\ of - - - -$

a) Constant magnitude b) constant direction c) zero magnitude d) equal magnitude

viii. $If\ \bar{u} \times \frac{d\bar{u}}{dt} = \bar{0}\ then\ the\ vector\ function\ \bar{u}(t) is\ of - - - -$

a) Constant magnitude b) constant direction c) zero magnitude d) equal magnitude

ix. Tangent vector to the curve $\bar{r}(t)$ *is* - - - - - - -

a) $\frac{d\bar{r}}{dt}$ b) $\left|\frac{d\bar{r}}{dt}\right|$ c) $\frac{d^2\bar{r}}{dt^2}$ d) $|\bar{r}(t)|$

x. $If\ \bar{v} = \bar{v}(x,y)\ \&\ x = x(s,t)\ ,\ y = y(s,t)\ then\ \frac{\partial \bar{v}}{\partial s} = - - -$

a) $\frac{\partial \bar{v}}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial \bar{v}}{\partial y}\frac{\partial y}{\partial s}$ b) $\frac{\partial \bar{v}}{\partial s} + \frac{\partial \bar{v}}{\partial s}$

c) $\frac{\partial \bar{v}}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial \bar{v}}{\partial t}\frac{\partial t}{\partial x}$ d) $\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{v}}{\partial y}$

***D) Numerical problems:***

i. Find $\bar{a}.\bar{b}$ if $\bar{a} = 2\bar{\imath} - 3\bar{\jmath} + \bar{k}$ & $\bar{b} = \bar{\imath} + \bar{\jmath} + \bar{k}$

ii. Find $|\bar{a} \times \bar{b}|$ if $\bar{a} = \bar{\imath} + \bar{\jmath} + \bar{k}$ & $\bar{b} = 2\bar{\imath} + 3\bar{\jmath} - \bar{k}$

iii. Find $\bar{a}.\bar{b} \times \bar{c}$ if $\bar{a} = \bar{\imath} + 2\bar{\jmath} - \bar{k}$ & $\bar{b} = 2\bar{\imath} - \bar{\jmath} - \bar{k}$ , $\bar{c} = 3\bar{\imath} - \bar{\jmath} - \bar{k}$

iv. $If\ \bar{f}(t) = \frac{sin3t}{t}\bar{\imath} + \frac{\log(1+t)}{t}\bar{\jmath} + \frac{3^t - 1}{t}\bar{k}\ ,for\ t \neq 0\ \&\ if\ \bar{f}\ is\ cotinuous\ at$ $t = 0\ , find\ \bar{f}(0).$

v. *Find the unit tangent vector of the vector* $\bar{r}(t) = sint\ \bar{\imath} - \cos t\bar{\jmath} + t\bar{k}.$

vi. $If\ \bar{a} = t\bar{\imath} + 2t\bar{\jmath} - t\bar{k}$ & $\bar{b} = t^2\bar{\imath} - 2t\bar{\jmath} + \bar{k}$ $find\ \frac{d}{dt}(\bar{a}.\bar{b})$

vii. $If\ \bar{a} = t\bar{\imath} + 2t\bar{\jmath} - t\bar{k}$ & $\bar{b} = t^2\bar{\imath} - 2t\bar{\jmath} + \bar{k}$ $find\ \frac{d}{dt}(\bar{a} \times \bar{b})$

viii. $If\ \bar{r} = (x^2 - 2y^2)\bar{\imath} + 5xy\bar{\jmath} + (2x^2y - x)\bar{k}\ find\ \frac{\partial \bar{r}}{\partial x}\ and\ \frac{\partial \bar{r}}{\partial y}$

ix. $If\ \bar{r} = (x^2 - 2y^2)\bar{\imath} + 5xy\bar{\jmath} + (2x^2y - x)\bar{k}\ find\ \frac{\partial^2 \bar{r}}{\partial x^2}$

x. $If\ \bar{r} = (x^2 - 2y^2)\bar{\imath} + 5xy\bar{\jmath} + (2x^2y - x)\bar{k}\ find\ \frac{\partial^2 \bar{r}}{\partial y^2}$

**Q.2 Theory Question (4-marks each)**

1. If $\bar{A}, \bar{B}, \bar{C}$ be any three vectors, then prove that $\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A}.\bar{C}) - \bar{C}(\bar{A}.\mathrm{B})$
2. If $\bar{A}, \bar{B}, \bar{C}$ be any three vectors, then prove that $(\bar{A} \times \bar{B}) \times \bar{C} = \bar{B}(\bar{A}.\bar{C}) - \bar{A}(\bar{B}.\bar{C})$
3. If $\bar{A}, \bar{B}, \bar{C}$ be any three vectors, then prove that
$\bar{A} \times (\bar{B} \times \bar{C}) + \bar{B} \times (\bar{C} \times \bar{A}) + \bar{C} \times (\bar{A} \times \bar{B}) = 0.$

4. If $\bar{A},\ \bar{B},\ \bar{C}$ be any three vectors, then prove that

   $(\bar{A}\times\bar{B}).(\bar{C}\times\bar{D}) = (\bar{A}.\bar{C})(\bar{B}.\bar{D}) - (\bar{A}.\bar{D})(\bar{B}.\bar{C})$ .

5. If $\bar{A},\ \bar{B},\ \bar{C}$ ,$\bar{D}$ be any four vectors, then prove that
   $(\bar{A}\times\bar{B})\times(\bar{C}\times\bar{D}) = \bar{C}[\bar{A}\ \ \bar{B}\ \ \bar{D}] - \bar{D}[\bar{A}\ \ \bar{B}\ \ \bar{C}] = \bar{B}[\bar{A}\ \ \bar{C}\ \ \bar{D}] - \bar{A}[\bar{B}\ \ \bar{C}\ \ \bar{D}]$
6. If $\bar{v}(t)$ is differentiable at $t = t_o$ then prove that it is continuous at $t = t_o$ , the converse is not true justify by contour example.
7. If $\bar{u}$ & $\bar{v}$ are differentiable functions of scalar variable t then prove that

   $\frac{d}{dt}(\bar{u}+\bar{v}) = \frac{d\bar{u}}{dt} + \frac{d\bar{v}}{dt}$

8. If $\bar{u}$ & $\bar{v}$ are differentiable functions of scalar variable t then prove that

   $\frac{d}{dt}(\bar{u}-\bar{v}) = \frac{d\bar{u}}{dt} - \frac{d\bar{v}}{dt}$

9. If $\bar{u}$ & $\bar{v}$ are differentiable functions of scalar variable t then prove that

   $\frac{d}{dt}(\bar{u}.\bar{v}) = \bar{u}.\frac{d\bar{v}}{dt} + \frac{d\bar{u}}{dt}.\bar{v}$

10. If $\bar{u}$ & $\bar{v}$ are differentiable functions of scalar variable t then prove that

    $\frac{d}{dt}(\bar{u}\times\bar{v}) = \bar{u}\times\frac{d\bar{v}}{dt} + \frac{d\bar{u}}{dt}\times\bar{v}$

11. If $\bar{u}$ is differentiable vector function of scalar variable t & $\phi$ is differentiable scalar function of scalar variable t then prove that $\frac{d}{dt}(\phi\,\bar{u}) = \bar{u}\frac{d\phi}{dt} + \phi\frac{d\bar{u}}{dt}$
12. If $\bar{u}$ *is differentiable vector* function of scalar variable s and s is differentiable scalar function of scalar variable t then *prove that* $\frac{d\bar{u}}{dt} = \frac{d\bar{u}}{ds}.\frac{ds}{dt} = \frac{ds}{dt}.\frac{d\bar{u}}{dt}$.
13. If $\bar{f}(t) = f_1(t)\bar{\imath} + f_2(t)\bar{\jmath} + f_3(t)\bar{k}$ is a differentiable vector function of the scalar variable t, then prove that $\frac{d}{dt}\bar{f}(t) = \frac{df_1(t)}{dt}\bar{\imath} + \frac{df_2(t)}{dt}\bar{\jmath} + \frac{df_3(t)}{dt}\bar{k}$
14. Prove a non-constant vector function $\bar{u}(t)$ *is of constant direction iff* $\bar{u}\times\frac{d\bar{u}}{dt} = 0$.

**Q.3 Examples** ***( 4- marks each )***

1. Find the value of $\bar{a}\times(\bar{b}\times\bar{c})$ , *if* $\bar{a} = \bar{\imath} - 2\bar{\jmath} + \bar{k}$ , $\bar{b} = 2\bar{\imath} + \bar{\jmath} + \bar{k}$ , $\bar{c} = \bar{\imath} + 2\bar{\jmath} - \bar{k}$
2. Find the value of $\bar{a}\times(\bar{b}\times\bar{c})$
   *if* $\bar{a} = 2\bar{\imath} - 10\bar{\jmath} + 2\bar{k}$ , $\bar{b} = 3\bar{\imath} + \bar{\jmath} + 2\bar{k}$ , $\bar{c} = 2\bar{\imath} + \bar{\jmath} + 3\bar{k}$.
3. If $\bar{a} = 2\bar{\imath} - \bar{\jmath} + 3\bar{k}$ , $\bar{b} = \bar{\imath} + \bar{\jmath} - 3\bar{k}$ , $\bar{c} = 3\bar{\imath} + 3\bar{\jmath} + 2\bar{k}$.

   *Verify that* $\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c}$

4. If $\bar{a} = 2\bar{\imath} - \bar{\jmath} + 3\bar{k}$ , $\bar{b} = \bar{\imath} + \bar{\jmath} - 3\bar{k}$ , $\bar{c} = 3\bar{\imath} + 3\bar{\jmath} + 2\bar{k}$.

   *Verify that* $\bar{b}\times(\bar{a}\times\bar{c}) = (\bar{b}.\bar{c})\bar{a} - (\bar{b}.\bar{a})\bar{c}$

5. If $\bar{a} = 2\bar{\imath} + 3\bar{\jmath} + 4\bar{k}$ , $\bar{b} = \bar{\imath} + \bar{\jmath} - \bar{k}$ , $\bar{c} = \bar{\imath} - \bar{\jmath} + \bar{k}$. *Find* $\bar{a} \times (\bar{b} \times \bar{c})$ and verify that $\bar{a} \times (\bar{b} \times \bar{c})$ is perpendicular to both $\bar{a}$ *and* $(\bar{b} \times \bar{c})$.
6. If $\bar{a} = 3\bar{\imath} + 2\bar{\jmath} - \bar{k}$ .Find $\bar{\imath} \times (\bar{a} \times \bar{\imath}) + \bar{\jmath} \times (\bar{a} \times \bar{\jmath}) + \bar{k} \times (\bar{a} \times \bar{k})$.
7. Show that $\bar{\imath} \times (\bar{a} \times \bar{\imath}) + \bar{\jmath} \times (\bar{a} \times \bar{\jmath}) + \bar{k} \times (\bar{a} \times \bar{k}) = 2\bar{a}$.
8. *Verify that* $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a}.\bar{c})\,\bar{b} - (\bar{a}.\bar{b})\,\bar{c}$ given that $\bar{a} = \bar{\imath} + 2\bar{\jmath} + 3\bar{k}$ , $\bar{b} = 2\bar{\imath} - \bar{\jmath} + \bar{k}$ , $\bar{c} = 3\bar{\imath} + 2\bar{\jmath} - 5\bar{k}$.
9. If $\bar{A} = \bar{\imath} + 2\bar{\jmath} - \bar{k}$ , $\bar{B} = 2\bar{\imath} + \bar{\jmath} + 3\bar{k}$ , $\bar{C} = \bar{\imath} - \bar{\jmath} + \bar{k}$. *and* $\bar{D} = 3\bar{\imath} + \bar{\jmath} + 2\bar{k}$

   Evaluate $(\bar{A} \times \bar{B}) . (\bar{C} \times \bar{D})$
10. If $\bar{A} = \bar{\imath} + 2\bar{\jmath} - \bar{k}$ , $\bar{B} = 2\bar{\imath} + \bar{\jmath} + 3\bar{k}$ , $\bar{C} = \bar{\imath} - \bar{\jmath} + \bar{k}$. *and* $\bar{D} = 3\bar{\imath} + \bar{\jmath} + 2\bar{k}$

    Evaluate $(\bar{A} \times \bar{B}) \times (\bar{C} \times \bar{D})$
11. If $\bar{A} = \bar{\imath} - 2\bar{\jmath} - 3\bar{k}$ , $\bar{B} = 2\bar{\imath} + \bar{\jmath} - \bar{k}$ , $\bar{C} = \bar{\imath} + 3\bar{\jmath} - 2\bar{k}$. *find that* $|(\bar{A} \times \bar{B}) \times \bar{C}|$
12. If $\bar{A} = \bar{\imath} - 2\bar{\jmath} - 3\bar{k}$ , $\bar{B} = 2\bar{\imath} + \bar{\jmath} - \bar{k}$ , $\bar{C} = \bar{\imath} + 3\bar{\jmath} - 2\bar{k}$. *find that* $|\bar{A} \times (\bar{B} \times \bar{C})|$
13. If $\bar{A} = \bar{\imath} - 2\bar{\jmath} - 3\bar{k}$ , $\bar{B} = 2\bar{\imath} + \bar{\jmath} - \bar{k}$ , $\bar{C} = \bar{\imath} + 3\bar{\jmath} - 2\bar{k}$. *find that* $(\bar{A} \times \bar{B}) \times (\bar{B} \times \overline{C})$
14. If $\bar{A} = \bar{\imath} - 2\bar{\jmath} - 3\bar{k}$ , $\bar{B} = 2\bar{\imath} + \bar{\jmath} - \bar{k}$ , $\bar{C} = \bar{\imath} + 3\bar{\jmath} - 2\bar{k}$. *find that* $(\bar{A} \times \bar{B}).(\bar{B} \times \overline{C})$
15. If $\bar{a} = 2\bar{\imath} + \bar{\jmath} - \bar{k}$ , $\bar{b} = -\bar{\imath} + 2\bar{\jmath} - 4\bar{k}$ , $\bar{c} = \bar{\imath} + \bar{\jmath} + \bar{k}$. *find* $(\bar{a} \times \bar{b}).(\bar{a} \times \bar{c})$
16. If $\bar{a} = \bar{\imath} + 2\bar{\jmath} - \bar{k}$ , $\bar{b} = 3\bar{\imath} - 4\bar{k}$ , $\bar{c} = -\bar{\imath} + \bar{\jmath}$ *and* $\bar{d} = 2\bar{\imath} - \bar{\jmath} - 3\bar{k}$, *find* $(\bar{a} \times \bar{b}).(\bar{c} \times \bar{d})$.
17. If $\bar{a} = \bar{\imath} + 2\bar{\jmath} - \bar{k}$ , $\bar{b} = 3\bar{\imath} - 4\bar{k}$ , $\bar{c} = -\bar{\imath} + \bar{\jmath}$ *and* $\bar{d} = 2\bar{\imath} - \bar{\jmath} - 3\bar{k}$, *find* $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})$.
18. If $\bar{a} = \bar{\imath} + \bar{\jmath} - \bar{k}$ , $\bar{b} = 2\bar{\imath} + \bar{\jmath} - 3\bar{k}$ , $\bar{c} = \bar{\imath} - \bar{\jmath} + 3\bar{k}$ & $\bar{d} = 3\bar{\imath} + 4\bar{\jmath} - 2\bar{k}$, *then find* $(\bar{a} \times \bar{b}).(\bar{c} \times \bar{d}) + (\bar{c} \times \bar{a}).(\bar{b} \times \bar{d}) + (\bar{d} \times \bar{a}).(\bar{c} \times \bar{b})$
19. Prove that $(\bar{A} \times \bar{B}) \times (\bar{C} \times \bar{D}) + (\bar{A} \times \bar{C}) \times (\bar{D} \times \bar{B}) + (\bar{A} \times \bar{D}) \times (\bar{B} \times \bar{C}) = -2[\bar{B}\ \ \bar{C}\ \ \bar{D}]\bar{A}$ .
20. Prove that $(\bar{B} \times \bar{C}).(\bar{A} \times \bar{D}) + (\bar{C} \times \bar{A}).(\bar{B} \times \bar{D}) + (\bar{A} \times \bar{B}).(\bar{C} \times \bar{D}) = 0$
21. *Prove that* $[\bar{a} \times \bar{b}\ \ \ \bar{b} \times \bar{q}\ \ \ \bar{c} \times \bar{r}] + [\bar{a} \times \bar{q}\ \ \ \bar{b} \times \bar{r}\ \ \ \bar{c} \times \bar{b}] + [\bar{a} \times \bar{r}\ \ \ \bar{b} \times \bar{p}\ \ \ \bar{c} \times \bar{q}] = 0$
22. Find a the set of vector reciprocal to the set of vectors $2\bar{\imath} + 3\bar{\jmath} - \bar{k}$ , $\bar{\imath} - \bar{\jmath} - 2\bar{k}$ , $-\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$.
23. Find a the set of vector reciprocal to the set of vectors $-\bar{\imath} + \bar{\jmath} + \bar{k}$ , $\bar{\imath} + \bar{\jmath} + \bar{k}$ , $\bar{\imath} + \bar{\jmath} - \bar{k}$.
24. Find a the set of vector reciprocal to the vectors $\bar{a}.\bar{b}$ *and* $\bar{a} \times \bar{b}$.
25. If $\bar{a}, \bar{b}, \bar{c}$ *is a set non − coplaner vectors and* $\overline{a'} = \frac{\bar{b} \times \bar{c}}{[\bar{a}\ \ \bar{b}\ \ \bar{c}]}$ $\overline{b'} = \frac{\bar{c} \times \bar{a}}{[\bar{a}\ \ \bar{b}\ \ \bar{c}]}$ & $\overline{c'} = \frac{\bar{a} \times b}{[\bar{a}\ \ \bar{b}\ \ \bar{c}]}$ *then prove that* $\bar{a} = \frac{\overline{b'} \times \overline{c'}}{[\overline{a'}\ \ \overline{b'}\ \ \overline{c'}]}$ , $\bar{b} = \frac{\overline{c'} \times \overline{a'}}{[\overline{a'}\ \ \overline{b'}\ \ \overline{c'}]}$ & $\bar{c} = \frac{\overline{a'} \times \overline{b'}}{[\overline{a'}\ \ \overline{b'}\ \ \overline{c'}]}$

26. $If\ \bar{a},\bar{b},\bar{c}\ \&\ \overline{a'}\ ,\overline{b'},\overline{c'}\ are\ reciprocal\ system\ of\ vectors\ then\ prove\ that$

$$\bar{a}\times\overline{a'} = \ \bar{b}\times\overline{b'} = \bar{c}\times\overline{c'} = \ \bar{0}$$

27. $If\ \bar{a},\bar{b},\ \bar{c}\ \&\ \overline{a'}\ ,\overline{b'},\overline{c'}\ are\ reciprocal\ system\ of\ vectors\ then\ prove\ that$

$$\overline{a'}\times\overline{b'} + \ \overline{b'}\times\overline{c'} + \overline{c'}\times\overline{a'} = \frac{\bar{a}+\bar{b}+\bar{c}}{[\bar{a}\quad\bar{b}\quad\bar{c}]}$$

28. $.If\ \bar{a},\bar{b},\ \bar{c}\ \&\ \overline{a'}\ ,\overline{b'},\overline{c'}\ are\ reciprocal\ system\ of\ vectors\ then\ prove\ that$

$$\bar{a}.\overline{a'} + \ \bar{b}.\overline{b'} + \bar{c}.\overline{c'} = \ 3$$

29. Evaluate $\lim_{t-0}\left[(t^2+1)\bar{\imath}+\left(\frac{3^{2t}-1}{t}\right)\bar{\jmath}+(1+2t)^{1/t}\bar{k}\right]$

30. If $\bar{f}(t)=\frac{\sin 2t}{t}\bar{\imath}+\cos t\,\bar{\jmath}\ ,\quad t\neq 0\ \ and\ \bar{f}(0)=x\bar{\imath}+\bar{\jmath}\ \ is\ continuos\ at$ $t=0\ ,find\ x.$

31. If $\bar{f}(t)=\frac{\sin 3t}{t}\bar{\imath}+\frac{\log(1+t)}{t}\bar{\jmath}+\ \frac{3^t-1}{t}\bar{k}\ ,\quad t\neq 0\ \ and\ \bar{f}\ is\ continuos\ at$ $t=0\ ,then\ find\ \bar{f}(0).$

32. $\bar{f}(t)=\cos t\,\bar{\imath}+\sin t\,\bar{\jmath}+\tan t\bar{k}\ ,\quad find\ \bar{f}'(t)\ \ and\ \ \left|\bar{f}'\left(\frac{\pi}{4}\right)\right|$

33. $If\ \ \bar{r}=(t^2+1)\bar{\imath}+(4t-3)\bar{\jmath}+(2t^2-6t)\bar{k},\ find\ \ \frac{d\bar{r}}{dt}\ \&\ \left|\frac{d\bar{r}}{dt}\right|\quad at\ \ t=2$

34. $If\ \ \bar{r}=(t^2+1)\bar{\imath}+(4t-3)\bar{\jmath}+(2t^2-6t)\bar{k},\ find\ \ \frac{d^2\bar{r}}{dt^2}\quad at\ t=2$

35. $If\ \ \bar{r}=(t^2+1)\bar{\imath}+(4t-3)\bar{\jmath}+(2t^2-6t)\bar{k},\ find\ \ \left|\frac{d^2\bar{r}}{dt^2}\right|$ at t=2

36. If $\bar{r}=e^{-t}\bar{\imath}+\log(t^2+1)\bar{\jmath}-\tan t\,\bar{k}\quad find\quad \left|\frac{d\bar{r}}{dt}\right|\quad at\ t=0$

37. If $\bar{r}=e^{-t}\bar{\imath}+\log(t^2+1)\bar{\jmath}-\tan t\,\bar{k}\quad find\quad \left|\frac{d^2\bar{r}}{dt^2}\right|$ at t = 0

38. If $\bar{a}=t^2\bar{\imath}+t\bar{\jmath}+(2t+1)\bar{k}\ and\ \ \bar{b}=(2t-3)\bar{\imath}+\bar{\jmath}-t\bar{k}\quad find\ \ \frac{d}{dt}(\bar{a}.\bar{b})$ at t = 1

39. If $\bar{a}=t^2\bar{\imath}+t\bar{\jmath}+(2t+1)\bar{k}\ and\ \ \bar{b}=(2t-3)\bar{\imath}+\bar{\jmath}-t\bar{k}\quad find\ \ \frac{d}{dt}\left|\bar{a}\times\bar{b}\right|\ \ at\ t=1$

40. If $\bar{a}=t^2\bar{\imath}+t\bar{\jmath}+(2t+1)\bar{k}\ and\ \ \bar{b}=(2t-3)\bar{\imath}+\bar{\jmath}-t\bar{k}\quad find\ \ \frac{d}{dt}\left(\bar{a}\times\frac{d\bar{b}}{dt}\right)at$ t = 1

41. If $\ \bar{u}=3t^2\bar{\imath}-(t+4)\bar{\jmath}+(t^2-2t)\bar{k}\ and\ \ \bar{v}\sin t\,\bar{\imath}+3e^{-t}\bar{\jmath}-3\cos t\,\bar{k}\ \ find$ $\frac{d^2}{dt^2}(\bar{u}\times\bar{v})\ at\ t=0$

42. $If\ \bar{r}=4asin^3\theta\ \bar{\imath}+4acos^3\theta\bar{\jmath}+3bcos2\theta\ \bar{k},find\ \ \left|\frac{d\bar{r}}{d\theta}\times\frac{d^2\bar{r}}{d\theta^2}\right|$

43. $If\ \bar{r}=4asin^3\theta\ \bar{\imath}+4acos^3\theta\ \bar{\jmath}+3bcos2\theta\ \bar{k},find\ \ \left[\frac{d\bar{r}}{d\theta}\quad\frac{d^2\bar{r}}{d\theta^2}\quad\frac{d^3\bar{r}}{d\theta^3}\right]$

44. $If\ \bar{r}=\bar{a}e^{2t}+\bar{b}e^{3t},prove\ that\ \ \frac{d^2\bar{r}}{dt^2}-5\frac{d\bar{r}}{dt}+\ 6\bar{r}=0.$

45. $Show\ that\ \bar{r}=e^{-t}\left(\bar{a}cos2t+\bar{b}sin2t\right),where\ \bar{a}\ \&\ \bar{b}\ are\ constant\ vectors\ is\ a$

$solution\ of\ the\ differential\ equation\ \ \frac{d^2\bar{r}}{dt^2}+2\frac{d\bar{r}}{dt}+5\bar{r}=0.$

46. $If\ \bar{r}=a\ cost\ \bar{\imath}+a\ sint\ \bar{\jmath}+at\ tan\alpha\ \bar{k},\ \ find\ \ \left|\frac{d\bar{r}}{dt}\times\frac{d^2\bar{r}}{dt^2}\right|$

47. $If\ \bar{r} = a\ cost\ \bar{\imath} + a\ sint\ \bar{\jmath} + at\ tan\alpha\ \bar{k},\ find\ \left[\frac{d\bar{r}}{dt}\ \frac{d^2\bar{r}}{dt^2}\ \frac{d^3\bar{r}}{dt^3}\right]$
48. $If\ \bar{r} = cosnt\ \bar{\imath} + sinnt\ \bar{\jmath},\ where\ n\ is\ constant\ show\ that\ \bar{r}.\frac{d\bar{r}}{dt} = 0$
49. $If\ \bar{r} = cosnt\ \bar{\imath} + sinnt\ \bar{\jmath},\ where\ n\ is\ constant\ show\ that\ \bar{r} \times \frac{d\bar{r}}{dt} = n\bar{k}.$
50. $If\ \bar{a} = sin\theta\ \bar{\imath} + cos\theta\ \bar{\jmath} + \theta\ \bar{k},\ \bar{b} = cos\theta\ \bar{\imath} - sin\theta\ \bar{\jmath} - 3\bar{k},\ \bar{c} = \bar{\imath} + 2\bar{\jmath} - 3\bar{k}$

    $find\ \frac{d}{d\theta}\left[\bar{a} \times (\bar{b} \times \bar{c})\right]\ at\ \theta = \frac{\pi}{2}$

51. Prove that $\frac{d}{dt}\left(\bar{r}.\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2}\right) = \bar{r}.\frac{d\bar{r}}{dt} \times \frac{d^3\bar{r}}{dt^3}$
52. $Show\ that\ \bar{r} = \bar{a}e^{kt} + \bar{b}e^{lt}\ is\ a\ solution\ of\ the\ differential\ equation\ q\bar{r} + p\frac{d\bar{r}}{dt} + \frac{d^2\bar{r}}{dt^2} = \bar{0}\ ,where\ k, l\ are\ roote\ of\ the\ equation\ m^2 + pm + q = 0,\ \bar{a}, \bar{b}\ \&\ p,$ $q\ being\ constant\ vectors\ \&\ scalars\ respectively.$
53. $If\ \bar{r} = a\ cost\ \bar{\imath} + a\ sint\ \bar{\jmath} + bt\ \bar{k}\ ,show\ that\ \left|\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2}\right|^2 = a^2(a^2 + b^2)$
54. $If\ \bar{r} = a\ cost\ \bar{\imath} + a\ sint\ \bar{\jmath} + bt\ \bar{k}\ ,show\ that\ \frac{d\bar{r}}{dt}\ .\frac{d^2\bar{r}}{dt^2} \times \frac{d^3\bar{r}}{dt^3} = a^2b.$
55. $If\ \bar{r} = \bar{a}\ cos\omega t + \bar{b}\ sin\omega t,\ where\ \bar{a}\ ,\ \bar{b}\ are\ constant\ vectors\ \&\ \omega\ is\ constant$

    $scalar, prove\ that\ \bar{r} \times \frac{d\bar{r}}{dt} = \omega(\bar{a} \times \bar{b})$

56. $If\ \bar{r} = \bar{a}\ cos\omega t + \bar{b}\ sin\omega t,\ where\ \bar{a}\ ,\ \bar{b}\ are\ constant\ vectors\ \&\ \omega\ is\ constant$

    $scalar, prove\ that\ \frac{d^2\bar{r}}{dt^2} = -\omega^2\bar{r}.$

57. Find the unit tangent vector & curvature at point P (x, y, z ) on the curve

    $\bar{r}(t) = 3\ cost\ \bar{\imath} + 3\ sint\ \bar{\jmath} + 4t\ \bar{k}.$

58. $Find\ the\ unit\ tangent\ vector\ \&\ the\ curvature\ at\ point\ P\ (x, y, z) on\ the\ curve$

    $x = a\ cos\theta\ , y = a\ sin\theta\ ,\ z = a\theta\ tan\alpha,\ where\ a\ \&\ \alpha\ are\ constants$

59. A particle moves along the curve $\bar{r} = e^{-t}\bar{\imath} + 2\ cos3t\ \bar{\jmath} + 2\ sin3t\ \bar{k}$, find the velocity & acceleration at any time t & also their magnitude at t = 0.
60. Find the velocity & acceleration vector of a particle moving along the curve x=2sin3t, y = 2 cos3t, z = 8t & also their magnitude.
61. Find the acute angle between the tangents to the curve $\bar{r} = t^2\bar{\imath} - 2t\bar{\jmath} + t^3\bar{k}$, at the points t = 1 & t = 2.
62. Show that the acute angle between the tangents to the curve x = t, y = $t^2$, z = $t^3$ at t = 1 & t = -1 is $\cos^{-1}\frac{3}{7}$.
63. Find the cosine of acute angle between the tangents to the curve $\bar{r} = t^2\bar{\imath} + 2t\bar{\jmath} - \frac{1}{2}t^2\bar{k}$ at t = 1 & t = -3.
64. Find the curvature of the curve $\bar{r} = a(3t - t^3)\bar{\imath} + 3at^2\bar{\jmath} + a(3t + t^3)\bar{k}.$

65. A particle moves along the curve x = $t^3 + 1$, y = $t^2$, z = 2t + 5, where t denotes time.

Find the component of its velocity in the direction $\bar{\imath} + \bar{\jmath} + 3\bar{k}$

66. A particle moves along the curve $\bar{r} = cost\,\bar{\imath} + sint\,\bar{\jmath} + t\,tan\alpha\,\bar{k}$, where $\alpha$ is constant & t is time variable. Find tangential & normal components of acceleration .
67. A particle moves along the curve $\bar{r} = (t^3 - 4t)\bar{\imath} + (t^2 + 4t)\bar{\jmath} + (8t^2 - 3t^3)\bar{k}$, where t is time variable. Find the tangential & normal components of acceleration at t = 2.
68. A particle moves along the curve $\bar{r} = 2t^2\bar{\imath} + (t^2 - 4t)\bar{\jmath} + (3t - 5)\bar{k}$, obtain the components of velocity & acceleration at t = 1 , along the direction $\bar{\imath} - 3\bar{\jmath} + 2\bar{k}$.
69. A particle moves along the curve $\bar{r} = e^t\bar{\imath} + e^{-t}\bar{\jmath} + \sqrt{2}\,t\bar{k}. Find\ i)\ \bar{v}\ \&\ \bar{a}\ \ ii)\ \bar{T}\ \&\ \bar{N}$ iii) the tangential & normal component of $\bar{a}$.
70. $If\ \bar{r} = x\,cosy\,\bar{\imath} + x\,siny\,\bar{\jmath} + ae^{my}\bar{k}, find\ i)\ \frac{\partial\bar{r}}{\partial x}\ \ ii)\ \frac{\partial\bar{r}}{\partial y}\ \ iii)\ \frac{\partial^2\bar{r}}{\partial x^2}\ \ iv)\ \frac{\partial^2\bar{r}}{\partial x\partial y}$
71. $If\ \bar{r} = cosxy\,\bar{\imath} + (3xy - 2x^2)\bar{\jmath} - (3x + 2y)\bar{k}, find\ i)\ \frac{\partial\bar{r}}{\partial x}\ ii)\ \frac{\partial\bar{r}}{\partial y}\ \ iii)\ \frac{\partial^2\bar{r}}{\partial x^2}\ \ iv)\ \frac{\partial^2\bar{r}}{\partial x\partial y}$
72. $If\ \bar{r} = x\,cosy\,\bar{\imath} + x\,siny\,\bar{\jmath} + c\log\left(x + \sqrt{x^2 - c^2}\right)\bar{k},\ \ find\ i)\ \frac{\partial\bar{r}}{\partial x}\ \ ii)\ \frac{\partial\bar{r}}{\partial y}$

$iii)\ \frac{\partial^2\bar{r}}{\partial y^2}\ \ iv)\ \frac{\partial^2\bar{r}}{\partial x\partial y}$

73. $\bar{r} = \frac{a}{2}(x + y)\bar{\imath} + \frac{b}{2}(x - y)\bar{\jmath} - \frac{xy}{2}\bar{k}\ \ ,\ find\ i)\ \frac{\partial\bar{r}}{\partial x}\ \ ii)\ \frac{\partial\bar{r}}{\partial y}\ \ iii)\ \frac{\partial^2\bar{r}}{\partial x^2}\ \ iv)\ \frac{\partial^2\bar{r}}{\partial x\partial y}$
74. $If\ \bar{r} = x\,cosy\,\bar{\imath} + y\,siny\,\bar{\jmath} + ae^{my}\bar{k}, find\ \frac{\frac{\partial\bar{r}}{\partial x}\times\frac{\partial\bar{r}}{\partial y}}{\left|\frac{\partial\bar{r}}{\partial x}\times\frac{\partial\bar{r}}{\partial y}\right|}$.
75. $If\ \bar{u} = x^2yz\,\bar{\imath} - 2xz^3\,\bar{\jmath} + xz^2\bar{k}$, and $\bar{v} = 2z\,\bar{\imath} + y\,\bar{\jmath} - x^2\bar{k},\ \ find$ $\frac{\partial^2}{\partial x\,\partial y}(\bar{u}\times\bar{v})\ at\ (1,\ 0,\ 2)$
76. $If\ \bar{r} = \frac{a}{2}(x + y)\bar{\imath} + \frac{b}{2}(x - y)\bar{\jmath} - xy\bar{k}\ \ find\ \ i)\ \left[\frac{\partial\bar{r}}{\partial x}\quad \frac{\partial\bar{r}}{\partial y}\quad \frac{\partial^2\bar{r}}{\partial x^2}\right]$

$ii)\ \left[\frac{\partial\bar{r}}{\partial x}\quad \frac{\partial\bar{r}}{\partial y}\quad \frac{\partial^2\bar{r}}{\partial x\partial y}\right]$

77. If $\bar{u} = z^3\,\bar{\imath} - x^2\bar{k}$, and $\bar{v} = 2xyz\,\bar{\jmath}, and\ \ \bar{w} = 5xy\,\bar{\imath} + 3z\,\bar{k}$

$find\ \frac{\partial^3}{\partial x\partial y\partial z}(\bar{u}\times\bar{v}.\bar{w})$.

78. If $\bar{r} = a\,cosu\,sinv\,\bar{\imath} + a\,sinu\,sinv\,\bar{\jmath} + acosv\bar{k}$ show that $\frac{1}{a}\left(\frac{\partial\bar{r}}{\partial u}\times\frac{\partial\bar{r}}{\partial v}\right)\quad is\ a\ unit\ vector$
79. If $\emptyset(x, y, z) = xy^2z\ \ \&\ \ \bar{u} = xz\bar{\imath} - xy^2\bar{\jmath} + yz^2\bar{k}$ find $\frac{\partial^3}{\partial^2x\partial^2z}(\emptyset\,\bar{u})at\ the\ point\ (2, -1, 1)$.
80. If $\bar{r} = e^{-\lambda x}\left(\bar{a}sin\lambda y + \bar{b}cos\lambda y\right)$ where $\bar{a}$ & $\bar{b}$ are constant vector & $\lambda$ is constant scalar .show that $\frac{\partial^2\bar{r}}{\partial x^2} + \frac{\partial^2\bar{r}}{\partial y^2} = 0$.

# Unit II

## (Differential Operators, Vector Integration)

**Q.1 Objective Questions** **( 2 marks each )**

***A) Fill in the blanks***

i) The gradient of a scalar point function is a - - - - - - function.

ii) The angle between two surfaces is defined as the angle between their - - - - - at the point of Intersection.

iii) The directional derivative of scalar point function $\varphi$ at point P along $\hat{a}$ is equal to - - - - - .

iv) The divergence of a vector point function is a - - - - - - - function.

v) A scalar point function which satisfies Laplace's equation is called - - - - - - -

vi)If $\bar{u}$ & $\bar{v}$ $are\ irrotational\ then\ \bar{u} \times\ \bar{v}\ is$ - - - - - - - -

vii) The curl of a vector point function is - - - - - - - -

viii) If C is a simple closed curve then the line integral of $\bar{f}$ over C is called --------- -

ix) Line integral may or may not depend upon - - - - - -

x) A vector field $\bar{f}$ about any closed curve in the region is - - - - - -

***B) Multiple choice Questions***

i) The dot product of two vectors is - - - - - -

a) Vector b) Scalar c) Vector field d) None of these

ii) The cross product of any vector with itself is – - - - -

a) $\bar{0}$ b) $\bar{1}$ $c)$ $\bar{2}$ d) None of these

iii) The angle between the vectors $(\bar{2\imath} + 3\bar{\jmath} + \bar{k})$ and $(\bar{2\imath} - \bar{\jmath} - \bar{k})$ is - - - - -

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{6}$

iv) If $|\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}|$ then the angle between $\bar{a}$ & $\bar{b}$ is-----------

$a)0^0$ $b)180^0$ $c)90^0$ $d)45^0$

v) The directional derivative of $\emptyset = xyz$ at (1, 1, 1) in the direction of $-\bar{j}$ is equal to ---------

a) 1 b) -1 c) 0 d) none of these

vi) If $\emptyset(x, y, z) = 2x^2y^3 - 3y^2z^3$ then $\nabla\emptyset$ at point (1, -1, 1) is equal to ---------------

$a) - 4\bar{\imath} + 12\bar{\jmath} + 9\bar{k}$ b) $4\bar{\imath} - 12\bar{\jmath} + 9\bar{k}$ c) $-4\bar{\imath} + 12\bar{\jmath} - 9\bar{k}$ d) none of these

vii) If $\bar{f} = x^2\bar{\imath} + y^2\bar{\jmath} + z^2\bar{k}$ then div.(curl$\bar{f}$) is equal to --------------

a) 1 b) 2 c) 0 d) none of these

viii) If $\nabla \times \bar{v} = \bar{0}$ then vector point function $\bar{v}$ is --------------

a) Solenoidal b) Irrotational c) Harmonic function d) none of these

ix) If $\bar{u} = t\bar{\imath} - t^2\bar{\jmath} + (t-1)\bar{k}$ and $\bar{v} = 2t^2\bar{\imath} + 6t\bar{k}$ then $\int_0^2 \bar{u}.\bar{v}\, dt$ is equal to -----------

a)12 b) 16 c) 14 d) none of these

x) The Value of $\int_0^{\pi} sin^2x\ dx$ is equal to ------------

a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

***C) Numerical Examples***

1) Find $\nabla r$ ,Where $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ and $|\bar{r}| = r$
2) Find the gradient of $x^2 + y^2 - z = 1$ at the point (1, 1, 1).
3) Find div$\bar{r}$, Where $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$
4) Find Curl$\bar{r}$, Where $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$
5) Find the directional derivative of f = xy + yz + zx in the direction of vector $\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$ at the point (1, 2, 0 )
6) *Find* $\nabla r^n$, Where $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$
7) $If\ \bar{f} = yz\bar{\imath} + zx\bar{\jmath} + xy\bar{k}\ \ then\ find\ div\bar{f}$
8) $If\ u = y^2 - z^2\ \ then\ find\ \nabla^2u$
9) $If\ \bar{a} = 2yz\bar{\imath} - x^2y\bar{\jmath} + xz^2\bar{k}\ and\ \varphi = 2x^2yz^3\ then\ find\ \bar{a}.\nabla\varphi$
10) $Evaluate\ \int_C \bar{f}.d\bar{r}\ ,where\ \bar{f} = 2xy\bar{\imath} + x^2\bar{\jmath}$ from (0 ,0) to (1, 1) along the straight line (0, 0) to (1, 1)

***D) Define the following***

1) Scalar point function

2) Vector point function

3) The Vector differential Operator ($\nabla$)

4) Gradient of a Scalar point function

5) Divergence of Vector point function

6) Solenoidal Vector function.

7) Curl of a Vector point function

8) Irrotational Vector function.

9) Line integral.

10) Conservative vector field.

**Q.2 Theory Questions** **( 4 marks each )**

1) *If* $\phi$ & $\psi$ *are scalar point functions* & *if* $\nabla\phi$ & $\nabla\psi$ exit in a given region R then prove that

   $\nabla(\phi \pm \psi) = \nabla\phi \pm \nabla\psi$ i.e. grad $(\phi \pm \psi) =$ grad $\phi \pm grad\ \psi$

2) Prove that a necessary & sufficient condition for a scalar point function $\phi$ to be constant is that $\nabla\phi = \bar{0}$.
3) If $\phi$ & $\psi$ are scalar point functions if $\nabla\phi$ & $\nabla\psi$ exist in a given region R then prove that $\nabla(\phi\psi) = \phi\,\nabla\Psi + \Psi\nabla\Phi$ i.e. grad $(\Phi\Psi) = \Phi\,\text{grad}\Psi + \Psi\,\text{grad}\Phi$
4) If $\phi$ & $\psi$ are scalar point functions if $\nabla\phi$ & $\nabla\psi$ exist in a given region R then prove that $\nabla\left(\frac{\phi}{\psi}\right) = \frac{\psi\,\nabla\phi - \phi\,\nabla\psi}{\psi^2}$ i.e. grad $\left(\frac{\phi}{\psi}\right) = \frac{\psi\,\text{grad}\phi - \phi\,\text{grad}\psi}{\psi^2}$ Provided $\psi \neq 0$
5) If $\phi$ (x, y, z) be a scalar point function defined in a region R. Let P(x, y, z) be a point in R & let $\hat{a}$ be a unit vector then prove that the directional derivative $\frac{d\phi}{ds}$ *of* $\phi$ *at P* along $\hat{a}$ *is given by* $\frac{d\phi}{ds} = \nabla\phi.\hat{a}$
6) If $\hat{n}$ be a unit vector normal to the level surface $\phi$ (x, y, z) = c at a point P of (x, y, z) in the direction of increment $\phi$ increasing & n be a distance along the normal , then prove that grad $\phi = \frac{d\phi}{dn}.\hat{n}$
7) If $\bar{u}$ & $\bar{v}$ be the vector point functions then prove that div $(\bar{u} \pm \bar{v})$ = div $\bar{u} \pm div\ \bar{v}$ i. e. $\nabla.(\bar{u} \pm \bar{v}) = \nabla.\bar{u} \pm \nabla.\bar{v}$
8) If $\bar{u}$ & $\bar{v}$ be the vector point functions then prove that curl $(\bar{u} \pm \bar{v})$ = curl $\bar{u} \pm curl\ \bar{v}$ i. e. $\nabla\times(\bar{u} \pm \bar{v}) = \nabla\times\bar{u} \pm \nabla\times\bar{v}$
9) If $\bar{u}$ be a vector point function & $\phi$ be any scalar point function then prove that Div $(\phi\bar{u})$ = (grad $\phi$). $\bar{u} + \phi\ div\ \bar{u}$ i. e. $\nabla.(\phi\bar{u}) = (\nabla\phi).\bar{u} \pm \phi(\nabla.\bar{u})$

10) If $\bar{u}$ be a vector point function & $\phi$ be any scalar point function then prove that
Curl $(\phi\bar{u})$ = (grad)$\times \bar{u} + \phi$ (curl $\bar{u}$) , i. e. $\nabla \times(\phi\bar{u}) = (\nabla\phi) \times \bar{u} + \phi(\nabla \times \bar{u})$
11) If $\bar{u}$ & $\bar{v}$ be the vector point functions then prove that div$(\bar{u} \times \bar{v}) = \bar{v}.curl\bar{u} - \bar{u}.curl\bar{v}$
i.e. $\nabla.(\bar{u} \times \bar{v}) = \bar{v}.(\nabla \times \bar{u}) - \bar{u}.(\nabla \times \bar{v})$
12) If $\bar{u}$ & $\bar{v}$ be the vector point functions then prove that
curl$(\bar{u} \times \bar{v}) = (\bar{v}.\nabla)\bar{u} - \bar{v}.div\bar{u} - (\bar{u}.\nabla)\bar{v} + \bar{u}$ div$\bar{v}$
i.e. $\nabla.(\bar{u} \times \bar{v}) = (\bar{v}.\nabla)\bar{u} - \bar{v}(\nabla.\bar{u}) + (\bar{u}.\nabla)\bar{v} + \bar{u}(\nabla.\bar{v})$
13) If $\bar{u}$ *and* $\bar{v}$ are two vector point functions then prove that
$grad(\bar{u}.\bar{v}) = (\bar{v}.\nabla)\bar{u} + (\bar{u}.\nabla)\bar{v} + \bar{v} \times (crul\bar{u}) + \bar{u} \times (crul\bar{v})$
i.e. $\nabla(\bar{u}.\bar{v}) = (\bar{v}.\nabla)\bar{u} + (\bar{u}.\nabla)\bar{v} + \bar{v} \times (\nabla \times \bar{u}) + \bar{u} \times (\nabla \times \bar{v})$
14) If $\varphi$ be a scalar point function then prove that
$curl\ (grad\ \phi) = \bar{0}$
i.e. $\nabla \times (\nabla\phi) = \bar{0}$
15) If $\bar{u}$ be a vector point function then prove that
Div .$(curl\bar{u}) = 0$
$i.e. \nabla.(\nabla \times \bar{u}) = 0$
16) If $\bar{u}$ be a vector point function then prove that
$Curl\ (curl\ \bar{u}) = grad(div\bar{u}) - \nabla^2\bar{u}$
i.e. $\nabla \times (\nabla \times \bar{u}) = \nabla(\nabla.\bar{u}) - \nabla^2\bar{u}$
17) If $\phi$ be a Scalar point function then prove that
$(\nabla.\nabla)\phi = \nabla(\nabla.\phi) = \nabla^2\phi$ where $\nabla^2$ be a Laplacian Operator
18) prove that a vector field $\bar{f}$ is conservative if and only if the circulation of $\bar{f}$ about any closed curve in the region is zero
19) Prove that if $\bar{f}$ be a continuously differentiable field on a region R then $\bar{f}$ is conservative if and only if it is the gradient of scalar point function defined on R(i.e. $\nabla\phi = \bar{f}$ )
20) Prove that if $\bar{f}$ be a continuously differentiable field on a region R then $\bar{f}$ is conservative if and only if it is irrotational (i.e. $\nabla \times \bar{f} = 0$ )

**Q.3 Examples** **(4 marks each)**

1. If $\bar{a} = xyz\bar{\imath} + xz^2\bar{\jmath} - y^3\bar{k}$ *and* $\bar{b} = x^3\bar{\imath} - xyz\bar{\jmath} + x^2z\bar{k}$ *then find*
$\frac{\partial^2\bar{a}}{\partial y^2} \times \frac{\partial^2\bar{b}}{\partial x^2}$ at the point (1, 1, 0).
2. If $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ and $|\bar{r}| = r$ *then Prove that*
i) $\nabla\phi(r) = \phi'(r)\nabla r$ ii) $\nabla r = \frac{\bar{r}}{r} = \hat{r}$
3. Prove that $\nabla r^n = nr^{n-2}\bar{r}$ *,where* $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$
4. Find grad (gradu.gardv) ,where $u = 3x^2y$ and $v = xz^2 - 2y$
5. Find $\phi(x, y, z)$if $\nabla\phi = 2xyz^3\bar{\imath} + x^2z^3\bar{\jmath} + 3x^2yz^2\bar{k}$ and $\phi(1, -2, 2) = 4$
6. Find the gradient and unit normal to the surface $x^2 + y^2 - z = 1$ at (1, 1 ,1)

7. Find the equation of tangent plane and the normal to the surface $xy + yz + zx = 7$ at the point (1, 1 ,3)
8. Find the acute angle between the tangents to surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1, -2, 1)
9. Find the cosine of the acute angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2)
10. Find the directional derivative of $\phi = 2xy + z^2$ at the point (1, -1, 3) in the direction of the vector $\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$
11. Find the directional derivative of $\phi(x, y, z) = x^2y + xz^2 - 2$ at the point A(1, 1, -1) along $\overline{AB}$ where B is the point (2, -1, 3)
12. Find the value of a & b if the surfaces $ax^2 - byz = (a + 2)x$ & $4x^2y + z^3 = 4$ are orthogonal at the point (1, -1, 2)
13. Find the value of the constant a, b, c so that the directional derivative of of $\phi(x, y, z) = axy^2 + byz + cz^2x^3$ at the point (1, 2, -1) as a maximum magnitude 64 in the direction parallel to Z-axis.
14. What is the greatest rate of increase of $u = xyz^2$ at (1, 0 ,3) ?
15. If $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ and $|\bar{r}| = r$ *then* find (i) div $(r^n\,\bar{r})$ , (ii) curl $(r^n\,\bar{r})$
16. If $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ and $|\bar{r}| = r$ *then* find Laplacian of $r^n$ i.e. $\nabla^2 r^n$
17. Prove that curl ($\phi$ grad$\phi$) = $\bar{0}$ where $\phi$ be any scalar point function.
18. Given that $\phi = 2x^3y^2z^4$ , find div ( grad $\phi$)
19. Determine the constant 'a' so that the vector function $\bar{v} = (x + 3y)\bar{\imath} + (y - 2z)\bar{\jmath} + (x + az)\bar{k}\, is\ solenoidal$
20. If the $\bar{f} = (axy - z^3)\bar{\imath} + (a - 2)x^2\bar{\jmath} + (1 - a)xz^2\bar{k}$ is irrotational then find the value of 'a'
21. Find the constant a, b, c so that the vector function $\bar{f} = (x + 2y + az)\bar{\imath} + (bx - 3y - z)\bar{\jmath} + (4x + cy + 2z)\bar{k}\ is\ irrotational$
22. If $\bar{\omega}\ is\ a\ constant\ vector$ & $\bar{v} = \bar{\omega} \times \bar{r}, prove\ that\ div(\bar{v}) = 0$
23. If $\bar{u}$ & $\bar{v}\ are\ irrotational\ the\ prove\ that\ \bar{u} \times \bar{v}\ is\ solenoidal$
24. Prove that $\bar{f} = f_1(y, z)\bar{\imath} + f_2(z, x)\bar{\jmath} + f_3(x, y)\bar{k}\ is\ solenoidal$
25. Prove that $\nabla\left[\frac{f(r)}{r}\,\bar{r}\right] = \frac{1}{r^2}\frac{d}{dt}(r^2 f(r))$
26. Prove that the vector function f(r) $\bar{r}\ is\ irrotational$
27. $If\ \bar{f} = t\bar{\imath} - 3\bar{\jmath} + 2t\bar{k}, \bar{g} = \bar{\imath} - 2\bar{\jmath} + 2\bar{k}$ & $\bar{h} = 3\bar{\imath} + t\bar{\jmath} - \bar{k}\ then\ evaluate$ $\int_1^2 \bar{f}.(\bar{g} \times \bar{h})dt$
28. $If\ \bar{f} = \sqrt{y}\bar{\imath} + 2x\bar{\jmath} + 2y\bar{k}, evaluate\ \int_C \bar{f}.d\bar{r}$ , where C is the curve given by

    $\bar{r} = t\bar{\imath} + t^2\bar{\jmath} + t^3\bar{k}$ from t = 0 to t = 1.

29. Find the total work done in moving a particle in a force field $\bar{f} = 3xy\bar{\imath} - 5z\bar{\jmath} + 10x\bar{k}$ Along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from t = 1 to t = 2

30. Evaluate $\int_C [(x^2 - y^2)\bar{\imath} + 2xy\bar{\jmath}].d\bar{r}$ around a rectangle with vertices at (0, 0), (a, 0), (a, b) & (0, b) transverse in the counter-clockwise direction.
31. Evaluate $\int_C \bar{f}.d\bar{r}, where\ \bar{f} = x^2\bar{\imath} + y^3\bar{\jmath}$ & C is the arc of the parabola $y = x^2$ in the XY plane from (0, 0) to (1, 1)
32. If $\bar{f} = 3xy\bar{\imath} - y^2\bar{\jmath}, evaluate\ \int_C \bar{f}.d\bar{r}$, where C is the curve in the XY plane $y = 2x^2$ from (0, 0) to (1, 2).
33. Evaluate $\int_C \bar{f}.d\bar{r}, where\ \bar{f} = yz\bar{\imath} + (zx+1)\bar{\jmath} + xy\bar{k}$ & C is any path from (1, 0, 0) to (2, 1, 4).
34. If $\bar{f} = (3x^2 + 6y)\bar{\imath} - 14yz\bar{\jmath} + 20xz^2\bar{k},\ evaluate\ \int_C \bar{f}.d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the paths (i) $x = t$, $y = t^2$, $z = t^3$ , (ii) the straight line joining (0, 0, 0) to (1, 1, 1).
35. Evaluate $\int(x\,dy - y\,dx)$ around the circle $x^2 + y^2 = 1$.
36. Find the circulation of $\bar{f}$ round the curve C, where $\bar{f} = y\bar{\imath} + z\bar{\jmath} + x\bar{k}$ & C is the circle $x^2 + y^2 = 1, z = 0$
37. If $\bar{u}(t) = t\bar{\imath} - t^2\bar{\jmath} + (t-1)\bar{k}$ & $\bar{v}(t) = 2t^2\bar{\imath} + 6t\bar{k}, then\ evaluate\ \int_0^2(\bar{u} \times \bar{v})dt$
38. The acceleration of a particle at any time t is given by $\bar{a} = e^{-t}\bar{\imath} - 6(t+1)\bar{\jmath} + 3sint\bar{k}.\ If\ the\ velocity\ \bar{v}$ & $displacement\ \bar{r}\ are\ zero\ at\ t = 0\ find\ \bar{v}$ & $\bar{r}\ at\ any\ time\ t$ .
39. Show that $\bar{f} = (2xy + z^3)\bar{\imath} + x^2\bar{\jmath} + 3xz^2\bar{k}$ is a conservative vector field. Find the scalar point ϕ function such that $\bar{f} = \nabla$

40. Determine whether the force field , $\bar{f} = 2xz\bar{\imath} + (x^2 - y)\bar{\jmath} + (2z - x^2)\bar{k}$ is conservative

or non- conservative .

# *Unit III*

## (Change of Axes, General Equation of Second Degree)

**Q.1 Objective Questions** **( 2 marks each )**

***(A) Fill in the blanks***

i. The two types of change of co-ordinate axes are - - - - - - & - - - - - - .
ii. The equations of translation are - - - - - & - - - - - - .
iii. If by rotation of axes, without change of origin, the expression $ax^2 + 2hxy + by^2$ Becomes $AX^2 + 2HXY + BY^2$ then the invariants are - - - - - - -.
iv. Parabola is a - - - - - conic (central/ non-central)
v. The general equation of a conic is - - - - - - .
vi. The centre of a conic given by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is - - - - - - -
vii. The conic given by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a parabola if - - - - - .
viii. In order to remove the xy terms from the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ the axes should be rotated through an angle - - - - -
ix. The equation $ax^2 + by^2 + 2gx + 2fy + c = 0$ represents a circle if - - - - - - .
x. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents - - - - - - when $ab – h^2 = 0$ & $hg – bg = 0$

***(B) Define the following:***

i. Transformation of co-ordinates
ii. Invariant
iii. Translation mapping
iv. Rotation mapping
v. A conic
vi. A parabola
vii. An ellipse
viii. A hyperbola
ix. Eccentricity
x. Directrix

*(C) Multiple choice questions*

i) If the origin is shifted to the point (2, 1), the directions of the axes remains the same then the equations of translation are

a) $x = 2 – X, y = 1 – Y$  b) $x = 1 – X , y = 2 – Y$

c) $x = 2 + X, y = 1 + Y$  d) $x = 1 + X, y = 2 + Y$.

ii) If by rotating the axes through an angle $45^0$ the equation $x^2 + 2xy + 5y^2 + 3x – 6y + 7 = 0$ becomes $px^2 + 2rxy + qy^2 + sx + ty + u = 0$ then the value of $pq – r^2$ is

a) 4  b) 6  c) 0  d) 5

iii) If the axes are rotated through an angle $45^0$ then the equations of rotation are

a) $x = \frac{X+Y}{\sqrt{2}}, y = \frac{X-Y}{\sqrt{2}}$  b) $x = \frac{X-Y}{\sqrt{2}}, y = \frac{X+Y}{\sqrt{2}}$

c) $x = \frac{X+Y}{2}, y = \frac{X-Y}{2}$  d) $x = \frac{X-Y}{2}, y = \frac{X+Y}{2}$

iv) A conic is a parabola if a) $e < 1$, b) $e > 1$, c) $e = 0$, d) $e = 1$

v) The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if

a) $h^2 – ab > 0$  b) $h^2 – ab < 0$  c) $h^2 – ab = 0$ d) $h^2 + ab > 0$

(Provided that $abc + 2fgh – af^2 – bg^2 – ch^2 \neq 0$ )

vi) The equation $16x^2 – 24xy + 9y^2 – 6x – 8y – 1 = 0$ represents

a) an ellipse b) a parabola c) a circle d) a hyperbola

vii) To remove the xy terms from the equation $7x^2 + 8xy + y^2 – 52x – 22y + 76 = 0$, $\tan 2\theta$ must be  a) $\frac{4}{3}$  b) $\frac{3}{4}$  c) 1  d) 0

viii)If a plane is perpendicular to an axis of the cone & cuts it , the section is

a) A parabola  b) an ellipse  c) a hyperbola  d) a circle

ix) The centre of the conic given by $x^2 – 4xy – 2y^2 + 10x + 4y = 0$ is at

a) (1, 2)  b) (1, -2)  c) (-1, 2)  d) ( -1, -2)

x) Length of the latus rectum for a conic $y2 = \frac{2}{5}x$ is a) $\frac{8}{5}$  b) $\frac{2}{5}$  c) $\frac{16}{5}$  d) $\frac{1}{10}$

***D) Numerical problems:***

i. Find the transformed form of the equation xy – x – 2y + 2 = 0 if the origin is shifted to the point (2, 1)

ii. If the axes are rotated through an angle $\theta = \sin^{-1}\frac{3}{5}$ keeping the origin fixed then find the equations of rotations.

iii. Find $\theta$ through which the axes should be rotated in order to remove the xy term from the equation $7x^2 + 12xy - 5y^2 + 4x + 3y - 2 = 0$.

iv. Find the centre of the conic given by the equation $5x^2 + 6xy + 5y^2 - 10x - 6y - 3 = 0$.

v. Identify the conic given by $16x^2 - 24xy + 9y^2 - 6x - 8y - 1 = 0$.

vi. Identify the conic given by $5x^2 - 6xy + 5y^2 + 18x - 14y + 9 = 0$.

vii. Through which angle the axes should be rotated to remove the xy term from $x^2 + 2xy + y^2 - 2x - 1 = 0$.

viii. Find the length of axes of the hyperbola $2y^2 - 3x^2 = 1$.

ix. Find the length of the latus rectum of the ellipse $2x^2 + 3y^2 = 6$.

x. If by rotation of axes keeping the origin fixed, the equation $x^2 + 2xy + 5y^2 + 3x - 6y + 7 = 0$ transform to $px^2 + 2rxy + qy^2 + sx + ty + u = 0$., then find the values of $p + q$ and $pq - r^2$ .

**Q.2 Theory Questions** ***( 6 marks each)***

i. Obtain the equations of translation when the origin is shifted to the point (h, k), directions of the axes remaining the same.

ii. Obtain the equations of rotations when the axes are rotated through an angle $\theta$ keeping the origin fixed.

iii. If by change of axes, without change of origin, the expression $ax^2 + 2hxy + by^2$ becomes $a'x'^2 + 2h'x'y' + b'y'^2$ then prove that $a + b = a' + b'$ & $ab - h^2 = a'b' - h'^2$.

iv. Show that equation of a conic is a second degree equation in x & y ( Hint: Use focus-directrix property).

v. If (x, y) & (x', y') are the co-ordinates of the same point referred to two sets of rectangular axes with the same origin & if ux + vy becomes u'x' + v'y' , where u & v are independent of x & y then show that $u^2 + v^2 = u'^2 + v'^2$.

vi. Show that if the set of rectangular axes is turned through some angle keeping the origin fixed then $g^2 + f^2$ in the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is invariant.

vii. Show that if the equations $ax^2 + 2hxy + by^2 = 1$ & $a'x'^2 + 2h'x'y' + b'y'^2 = 1$ represent the same conic & if the axes are rectangular then prove that $(a - b)^2 + 4h^2 = (a' - b')^2 + 4h'^2$. ( Hint: use $a + b = a' + b'$ & $ab - h^2 = a'b' - h'^2$).

viii. If by rotation of the axes the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ becomes $a'x'^2 + 2h'x'y' + b'y'^2 + 2g'x' + 2f'y' + c' = 0$ then prove that

$$a' = \frac{1}{2}\left[(a+b) + \sqrt{4h^2 + (a-b)^2}\right] \ \ \& \ b' = \frac{1}{2}\left[(a+b) - \sqrt{4h^2 + (a-b)^2}\right]$$

(Hint: Use $a' = \frac{1}{2}\,[(a+b)+(a-b)\cos 2\theta + 2h\sin 2\theta]$,
$b' = \frac{1}{2}\,[(a+b)-(a-b)\cos 2\theta - 2h\sin 2\theta]$ & then use $\tan 2\theta = \frac{2h}{a-b}$)

ix. Prove that every general equation of second degree represents a conic.

***Q.3 Examples*** ***(4 marks each)***

1) If the origin is shifted to the point (h, 2), find the value of 'h' so that the new equation of the locus given by the equation $x^2 + 4x + 3y - 5 = 0$ will not contain a first degree terms in x.
2) The origin is shifted to the point (-2, k), find the value of 'k' so that the new equation of the locus given by $2y^2 + 3x + 4y - 7 = 0$ will not contain the first degree term in y.
3) Obtain the new equation of the locus given by $3x^2 + y^2 + 18x - 8y - 16 = 0$ when the origin is shifted to the point (-3, 4).
4) Obtain the transformed equation of the locus given by $3x^2 + 2\sqrt{3}\,xy + 5y^2 = 1$ when the axes are rotated through an angle of $60^0$.
5) Transform the equation $3x^2 + 2xy + 3y^2 + 8x + 3y + 4 = 0$ by rotating the axes through an angle $\theta$, where $\theta = \sin^{-1}\frac{3}{5}$ , $0 < \theta < \frac{\pi}{2}$, keeping the origin fixed.
6) Find the new equation of the locus given by $x^2 + 3y^2 + 4x + 18y + 30 = 0$ when the origin is shifted to the point (-2, -3) directions of the axes remaining the same.
7) If the axes are turned through an angle of $45^0$ keeping the origin fixed, show that the equation of the locus given by $x^2 - 4xy + y^2 = 0$ changes to $3y'^2 - x'^2 = 0$.
8) The equation of the curve referred to the axes through (-1, 2) as origin & parallel to the original axes is $2X^2 + 3Y^2 = 6$. Find the equation of the curve referred to original set of axes.
9) What does the equation $3x^2 - 4xy + 25y^2 = 0$ become when the axes are rotated through an angle $\tan^{-1} 2$ ?
10) Find the co-ordinates of a point to which the origin should be shifted so that the new equation of the locus given by $x^2 - 2xy + 3y^2 - 10x + 22y + 30 = 0$ will not contain the first degree terms in the new co-ordinates.
11) The axes are changed by changing the origin to $(\alpha, 2)$ .By this transformation the line given by $x + 2y + 3 = 0$ passes through the origin .Find the value of $\alpha$.
12) Transforms the equation of a circle $x^2 + y^2 + 2x + 2y + 1 = 0$ to standard form.
13) Transform the equation $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$ when the origin is shifted to the point (-1, 1) and then the axes are turned through an angle of $45^0$.
14) What does the equation $x^2 - 5xy + 13y^2 - 3x + 21y = 0$ when the origin is changed to (-1, -1) and then the axes turned through an angle $\tan^{-1}\left(\frac{1}{5}\right)$.
15) Transform the equation $7x^2 - 8xy + y^2 + 14x - 8y - 2 = 0$ when the origin is shifted to the point (-1, 0) and then the axes are turned through an angle of $\tan^{-1}\left(\frac{-1}{2}\right)$

16) The equation $3x^2 + 2xy + 3y^2 - 18x - 22y + 50 = 0$ transforms to $4x'^2+2y'^2=1$ when the origin is shifted to the point (2, 3) and then the axes are rotated through an angle $\theta$.Find the measure of an angle $\theta$
17) Change the origin to (1, 2) and transform $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$.Further rotate the axes through $\theta = \frac{\pi}{4}$ and find the final transform of the equation.
18) Transform the equation $11x^2 + 24xy + 4y^2 - 20x - 40y – 5 = 0$ to rectangle axes through the point (2, -1) inclined at an angle $\tan^{-1}\left(\frac{-4}{3}\right)$ to the original axes.
19) Transform the equation $5x^2 + 6xy + 5y^2 - 10x - 6y – 3 = 0$ when the origin is changed to (1, 0) and then the axes are rotated through an angle $\left(\frac{-\pi}{4}\right)$.
20) Obtain the equation of rotation in order to remove the xy term form $x^2 + 6xy + 8y = 7y^2 + 8x + 20$.
21) Find the centre of a curve and identify it .$3x^2 + 8xy - 7y^2 – x + 7y – 2 = 0$
22) Find the centre of the following conics and identify each of them $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$
23) Find the centre of the following conics and identify each of them $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$
24) Find the centre of the following conics and identify each of them $5x^2 + 6xy + 5y^2 - 10x - 6y – 3 = 0$
25) Find the centre of the following conics and identify each of them $55x^2 - 30xy + 39y^2 - 40x - 24y – 464 = 0$
26) Find the centre of the following conics and identify each of them $8x^2 - 24xy + 15y^2 + 48x - 48y + 7 = 0$
27) Reduce the equation of a parabola $16x^2 - 24xy + 9y^2 - 6x - 8y – 1 = 0$ in the standard form
28) Find the centre of a conic $7x^2 + 8xy + y^2 - 52x - 22y + 76 = 0$ and reduce it to its standard form.
29) Transform the equation of a conic $x^2 - 4xy - 2y^2 + 10x + 4y = 0$ to its standard form.
30) Find the co-ordinate of the centre of the conic $5x^2 + 6xy + 5y^2 - 4x - 4y – 4 = 0$ and reduce the equation of the conic to its standard form.
31) Transform the equation $3(x^2 + y^2 + 1) = 2y (12x + 1) - 14x (y + 1)$ to the form $\alpha x^2 + \beta y^2 = 1$
32) Show that the equation $x^2 + 2xy + y^2 - 2xy – 1 = 0$ represented a parabola .Reduce the equation to its standard form .Also find the length of the latus rectum.
33) Show that the equation $x^2 - 4xy - 2y^2 + 10x + 4y = 0$ represent a hyperbola .Find its centre .Also find the equation of the asymptotes.
34) Find the centre of the conic $5x^2 - 6xy + 5y^2 + 18x - 14y + 9 = 0$ and reduce it to the standard form .Also find the eccentricity of the conic
35) Determine the nature of the following conics. Also find the centre and length of axes in each case,$5x^2 + 6xy + 5y^2 - 10x - 6y – 3 = 0$

36) Determine the nature of the following conics. Also find the centre and length of axes in each case $36x^2 + 24xy + 29y^2 - 72x + 126y + 81 = 0$
37) Determine the nature of the following conics. Also find the centre and length of axes in each case $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$
38) Determine the nature of the following conics. Also find the centre and length of axes in each case $9x^2 + 24xy + 16y^2 - 44x + 108y - 124 = 0$
39) Determine the nature of the following conics. Also find the centre and length of axes in each case $32x^2 + 52xy - 7y^2 - 64x - 52y - 148 = 0$
40) Determine the nature of the following conics. Also find the centre and length of axes in each case $16x^2 - 24xy + 9y^2 - 104x - 172y + 44 = 0$.

# Unit IV

## (Sphere)

**Q.1 Objective questions** **( 2 marks each )**

***A) Fill in the blanks:***

i. A - - - - - - is the locus of a point which moves in a space so that it is always at a constant distance from a fixed point.
ii. Equation of a sphere is - - - - - - - degree equation in x, y, z.
iii. The intersection of the sphere & the plane is - - - - - - - - .
iv. A line which meets a sphere in two coincident points is called the - - - - - - line to the sphere.
v. The locus of the tangent lines to a sphere at a point on it is called the - - - - - - plane at that point.
vi. The plane is tangent plane to the sphere iff length of the perpendicular from the centre is equal to - - - - - - -.
vii. Two spheres are said to cut orthogonally if they intersect each other at - -- - - - - angles.
viii. Two spheres are non-intersecting if distance between the centres is greater than the sum of - - - - - - - of the spheres.
ix. Two spheres touch each other externally if distance between the centres is equal to the sum of the - - - - - - of the spheres.
x. The plane of the great circle passes through the - - - - - - of the sphere.

***B) Define the following***:

i. Sphere
ii. Tangent to the sphere.
iii. Tangent plane to the sphere.
iv. Normal to the sphere at a point.
v. Great circle.
vi. State condition of tangency.
vii. Orthogonal sphere.
viii. State condition of orthogonality.
ix. State general equation of the sphere.
x. State equation of sphere having centre at origin & radius is a.

***C) Numerical problems:***

i. Find the centre of the sphere $2x^2 + 2y^2 + 2z^2 + 3x + 4y – 6z – 4 = 0$.
ii. Find the radius of the sphere $2x^2 + 2y^2 + 2z^2 + 3x + 4y – 6z – 4 = 0$.
iii. Find the centre & radius of the sphere $x^2 + y^2 + z^2 + 4x – 6y – 8z = 2$.
iv. Find the radius of the sphere passing through the point (2, 1, 3) & having the centre at (1, -3, 4).

v. Find the centre of the sphere described on (2, -3, 1) & (3, -1, 2) as extremities of a diameter.

vi. Find the radius of the sphere described on (2, -3, 1) & (3, -1, 2) as extremities of a diameter.

vii. Find the equation of the sphere having centre at (1, 2, 3) & radius 3.

viii. Find the equation of the sphere having centre at origin & radius 4.

ix. Find the equation of the sphere whose centre is (-1, 7, 3) & which passes through the origin.

x. Find the centre of the great circle in the sphere $x^2 + y^2 + z^2 + 4x - 6y + 4z - 8 = 0$.

***D) Multiple choice questions:***

i. The general equation of the sphere is
a) Linear b) second degree c) third degree d) none of these

ii. The section of the sphere taken by the plane is
a) Sphere b) circle c) plane d) none of these

iii. The centre of the great circle & the corresponding sphere are
a) Different b) same c) not repeated to each other d) none of these

iv. The number of tangent lines at a point on the sphere are
a) Two b) three c) infinite d) none of these.

v. The tangent plane to the sphere touches the sphere at
a) One point b) two point c) three point d) none of these.

vi. The normal line to the sphere at a point passes through
a) Centre of the sphere c) tangent plane
b) Great circle d) none of these

vii. If equation $S + \lambda S' = 0$ represents a radical plane then
a) $\lambda = 1$ b) $\lambda = -1$ c) $\lambda = 0$ d) none of these.

viii. The angle between the tangent planes at a common point of the orthogonal spheres is
a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) 0 d) none of these.

ix. The radius of the spheres $x^2 + y^2 + z^2 + 4x - 6y - 8z = 2$ is
a) $\sqrt{32}$ b) $\sqrt{31}$ c) 31 d) none of these.

x. If the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ passes through origin then
a) d= -1 b) d= 0 c) d= 1 d) none of these

**Q.2 Theory Questions (6- marks each)**

1. Show that the equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a sphere .Find the centre and radius.
2. Obtains the equation of the sphere which passes through the origin and makes intercepts a, b, c on co-ordinate axes .Hence find the equation of the sphere passing through the origin and making intercept 2, 3, 4 on the axes .
3. Find the equation of the sphere with line joining the points A $(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ as one the diameters. Hence obtain the equation of the sphere described on (2, -3, 1) and (3, -1, 2) as extremities of a diameter.

4. Find the condition that the plane lx + my+ nz = p may touches the sphere $x^2 + y^2 + z^2 = a^2$ and find the point of contact.
5. Find the condition that the plane lx + my+ nz = p may touches the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$
6. Find the equation of the sphere passing through the four points $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$, $R(x_3, y_3, z_3)$ and $S(x_4, y_4, z_4)$.
7. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ at the point $A(\alpha, \beta, \gamma)$.
8. Obtain the equation of the normal to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ at the point $A(\alpha, \beta, \gamma)$.
9. Let the equation of a circle be $S = x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ and $U = lx + my + nz - p = 0$. Show that $S + \lambda U = 0$, where $\lambda$ is a parameter represents a family of sphere through the given circle.
10. Define orthogonal spheres. Obtain the contain that the spheres $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = {}^0$ are orthogonal to each other.
11. Define a sphere. Obtain the equation of the sphere whose centre is (a, b, c) and radius is r and states the characteristics of the equation of the sphere.

**Q.3 Examples** **(4- marks each)**

1. Find the equation of the sphere passing through O (0, 0, 0), A (a, 0, 0), B (0, b, 0), C(0, 0, c)
2. Find an equation of the sphere with the centre at (1,-3, 4) and passing through the point (2, 1, 3).
3. Find the equation of the sphere which passes through the points (2, 4, -1), (0, -4, 3) , (-2, 0, 1) and (6, 0, 9 ).
4. Find the equation of the sphere which passes through the points (1, 2, 3), (0, -2, 4), (4, -4, 2) and (3, 1, 4).
5. Find the equation of the sphere which passes through the points A(1, 0, 0), B(0, 1, 0), C(0, 0, 1) and has its radius as small as possible .
6. Find the equation of the sphere described on (2,-3, 1) and (3, -1, 2) as extrimities of a diameter.
7. Find the equation of the sphere with centre at (-1, 2, 3) , and passing through the point (1, -1, 2).
8. A plane passes through a fixed point (a, b, c) .Show that the locus of the foot of the perpendicular to it from the origin is the sphere.$x^2 + y^2 + z^2 - ax - by - cz = 0$.
9. Find the equation of the sphere passing through the points (1, 2, 3), (0, -2, 4), (4, -4, 2) and having its centre on the plane 2x - 5y - 2z - 5 = 0 .
10. Find the equation of the sphere passing through the points A (3, 0, 2), B(-1, 1, 1), C (2, -5, 4) and having its centre on the plane 2x + 3y + 4z - 6 = 0
11. Find the equation of the sphere passing through the points (0, 0, 0), (-1, 2, 0), (0, 1, -1) and (1, 2, 3).

12. A sphere of radius k passes through the origin and meets the axes in A, B, C .Prove that the locus of the centroid of the triangle ABC is the sphere $g( x^2 + y^2 + z^2 ) = 4k^2$.
13. Find the co-ordinate of the centre and radius of the circle $x^2 + y^2 + z^2 - 2y - 4z = 11$, $x + 2y + 2z = 15$.
14. Find the centre and radius of the circle $x^2 + y^2 + z^2 – 2x – 4y - 6z – 2 = 0$, $x + 2y + 2z = 20$.
15. Obtain the equation of the sphere through the three points (1, -1, 1), (3, 3, 1), (-2, 0 ,5) and having its centre on the plane $2x – 3y + 4z – 5 = 0$.
16. Find the co-ordinates of the points of intersection of the line and the sphere

    $\frac{x+2}{4} = \frac{y+9}{3} = \frac{z-8}{-5}$ , $x^2 + y^2 + z^2 = 49$

17. Show that the plane $2x – 2y + z + 16 = 0$ touches the sphere $x^2 + y^2 + z^2 + 2x – 4y + 2z – z = 0$ and find the co-ordinate of the points of contact.
18. Show that the line $\frac{x-6}{3} = \frac{y-7}{4} = \frac{z-3}{5}$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4z – 4 = 0$. Also find the co-ordinates of the point of contact.
19. Find the equation of the tangent plane to the sphere $3(x^2 + y^2 + z^2) – 2x - 3y - 4z - 22 = 0$ at the point (1, 2, 3).
20. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 – 2x - 10z – 9 = 0$ at the point (4, 5, 6).
21. Find the equation of the normal plane to the sphere $x^2 + y^2 + z^2 – 2x - 10z - 9 = 0$ at the point (4, 5, 6).
22. Show that the plane $2x – 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 – 2x - 4y + 2z = 3$ and the point of contact.
23. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 – 2x - y - z -5 = 0$ at the point (1, 1, -2).
24. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 – 4 = 0$, $2x + 4y + 6z – 1 = 0$ and having its centre on the plane $x + y + z = 0$
25. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 + 10y – 4z – 8 = 0$, $x + y + z = 3$ as a great circle.
26. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 – 3x + 4y - 2z - 5 = 0$, $5x - 2y + 4z + 7 = 0$ as a great circle.
27. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 – 6x - 4y + 10z = 0$ at the origin
28. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ as a great circle .
29. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 = 5$ , $x + 2y + 3z = 3$ and touch the plane $4x + 3y = 15$ .
30. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 - 2x – 4y = 0$, $x + 2y + 3z = 8$ and touch the plane $4x + 3y = 25$.
31. Show that the sphere $x^2 + y^2 + z^2 = 25$ and $x^2 + y^2 + z^2 - 24x – 40y - 18z + 225 = 0$ touch each other externally .
32. Show that the sphere $x^2 + y^2 + z^2 + 4y – 5 = 0$ and $x^2 + y^2 + z^2 – 6y + 5 = 0$ touch each other externally and find the co-ordinate of the point of contact.

33. Show that the sphere $x^2 + y^2 + z^2 = 64$ and $x^2 + y^2 + z^2 – 12x + 4y – 6z + 48 = 0$ touch each other externally and find the co-ordinate of the point of contact.
34. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 + 4x –2y + 4z – 16 = 0$, $2x +2y + 2z + 9 = 0$ and a given point (-3, 4, 0 ).
35. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and a given point (1, 2, 3).
36. Show that the sphere $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 8y + 4z + 20 = 0$ cut orthogonally.
37. Show that the sphere $x^2 + y^2 + z^2 + 7x + 10y - 5z + 12 = 0$ and $x^2 + y^2 + z^2 – 4x + 6y + 4 = 0$ intersect orthogonally
38. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 + x – 3y + 2z - 1 = 0$ , $2x + 5y – z + 7 = 0$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 3x + 5y - 7z - 6 = 0$
39. Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0$ , $3x - 4y + 5z - 15 = 0$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$
40. Show that the sphere $x^2 + y^2 + z^2 – 14x + 45 = 0$ and $x^2 + y^2 + z^2 + 4x - 117 = 0$ touch each other externally and find the co-ordinate of the point of contact.

# Unit V

## (Cone, Cylinder and Conicoids)

**Q.1 Objective questions** **(2 marks each)**

***A) State true or false and justify your answer.***

1. Equation of cone with vertex at origin is non homogenous.
2. Every homogenous equation of degree in x, y, z represents a cone with vertex at the origin.
3. If a, b, c are the direction ratio of any generator of the cone f(x, y, z) = 0 with vertex origin then f (a, b, c) = 0.
4. If the number a, b, c satisfy equation of the cone with vertex origin then a, b, c are the direction ratios of some generator of that cone.
5. A section of a right circular cone by a plane perpendicular to its axis and not passing through the vertex is a circular.
6. The point (1, 1, 1) the vertex of the cone $5x^2 + 3y^2 + z^2 - 2xy - 6yz - 4zx + 6x + 8y + 10z - 26 = 0$.

***B) Fill in the blanks***

1. The equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$ *is* $5x^2 + 8y^2 + 5z^2 - 4xy + (--) - 8zx + (--) - 16y - 14z + (--) = 0$
2. The equation of the tangent plane at the point P (-2, 2, 3) to the ellipsoid $4x^2 + y^2 + 5z^2 = 65$ is $8x + (---) - 15z + (----) = 0$
3. The equation of the tangent plane at the point P (-2, 2, 3)to the ellipsoid $4x^2 + y^2 + 5z^2 = 65$ is $8x - 2y - 15z - 65 = 0$ then the normal at point is $\frac{x+2}{--} = \frac{y-2}{--} = \frac{--}{-15}$
4. The equation of a right circular cone with vertex (2, 1, -3) whose axis parallel to Y-axis & semi vertical angle $45^0$ is $x^2 + (---) + z^2 - 4x + (---) + (---) + (----) = 0$ .

***C) Define the following terms.***

1. Cone & Guiding curve
2. Quadric cone
3. Right circular cone
4. Enveloping cone of the sphere
5. Cylinder
6. Right circular cylinder
7. Enveloping cylinder
8. Tangent line & tangent plane
9. Director sphere
10. Normal at point & foot of the normal

***D) Multiple choice Questions:***

1. (A) Every homogeneous equation of degree in x, y, z represents a cone with vertex at the origin (B) If the numbers a, b, c satisfy equation of the cone with vertex at origin then a, b, c are the direction ratios of some generator of that cone.

   a) A is false, B is false b) A is true , B is true,

   c) A is false , B is true, d) A is true, B is false

2. $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ this is equation of
   a) Quadratic cone with vertex at origin c) right circular cone
   b) cylinder when guiding curve is on XY plane d) cone with vertex at $(\alpha, \beta, \gamma)$
3. $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$, this equation represents
   a) Ellipsoid b) hyperboloid of one sheet c) hyperboloid of two sheet d) none of these
4. The equation $5x^2 + 3y^2 + z^2 - 2xy - 6yz - 4xz + 6x + 8y + 10z - 26 = 0$ represent cone then vertex of this cone is a) (3, 1, 2) b) (1, 2, 3) c) (2, 3, 1) d) (2, 1, 3)
5. The line $\frac{x-3}{2} = \frac{y-3}{3} = \frac{z+3}{-4}$ *touches ellipsoid* $\frac{x^2}{8} + \frac{y^2}{9} + \frac{z^2}{4} = 1$ then the point of contact is a) $(\frac{2}{3}, \frac{1}{2}, 2)$ b) $\left(2, \frac{2}{3}, \frac{1}{2}\right)$ c) $(2, \frac{3}{2}, 1)$ d) none of these
6. The vertex of the cone $4x^2 + 3y^2 - 5z^2 - 6yz - 8x + 16z - 4 = 0$ is
   a) (1, 1, 1) b) (2, 2, 2) c) (1, 2, 3) d) none of these
7. (A) $\frac{l^2}{A} + \frac{m^2}{B} + \frac{n^2}{C} = p$ is condition that the plane lx + my + nz = p is tangent plane to the conicoid $Ax^2 + By^2 + Cz^2 = 1$ & (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$, this equation represents hyperboloid of one sheet
   a) A is true, B is true c) A is true, B is false
   b) A is false, B is true d) A is false, B is true
8. The plane 6x – 5y – 6z = 20 touches the hyperboloid $4x^2 - 5y^2 + 6z^2 = 40$ then the point of contact is
   a) (3, 2, 2) b) (3, -2, -2) c) (-3, 2, 2) d) (3, 2, -2)
9. The plane 7x + 5y + 3z = 30 touches the ellipsoid $7x^2 + 5y^2 + 3z^2 = 60$ then the point of contact is
   a) (1, 1, 1) b) (2, 2, 2) c) (3, 3, 3) d) ( 4, 4, 4)
10. The plane 3x + 12y – 6z = 17 touches the conicoid $3x^2 - 6y^2 + 9z^2 + 17 = 0$ then the point of contact is

    a) (1, 2, 3) b) (-1, 2, 3) c) (-1, 2, 2/3) d) (1, 2, 2/3)

***E) Numerical problems:***

1. Define quadric cone , write down the general equation of quadric cone with vertex at the origin
2. State the general equation of a cone with vertex at the point $V(\alpha, \beta, \gamma)$

3. Write down the condition that the general equation of the second degree should represent a cone.
4. Write down the equation of right circular cone whose vertex at origin, semi vertical angle $\theta$ & direction ratios of the axis a, b, c.
5. Write the equation of the right circular cone whose vertex at $(\alpha, \beta, \gamma)$, semi vertical angle & direction cosine of axis are l, m, n.
6. State the equation of right circular cone whose vertex at origin, Z axis as the axis, semi vertical angle.
7. Write down the condition that the plane is tangent plane to the conicoid & also write the co-ordinate of point of contact.
8. Write down the length of the perpendicular P on tangent plane $Axx_1 + Byy_1 + Czz_1 = 1$ from the origin also writes the direction cosine of the normal to the above plane.
9. If the conicoid is ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ write down the equation of the normal at $(x_1, y_1, z_1)$ in terms of direction cosines.
10. Write down the equation of right circular cylinder whose axis is the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ & whose radius is r.

**Q.2 Theory Questions: ( 4 marks each )**

1. Show that the equation of the cone with vertex at the origin is homogeneous.
2. Show that every homogeneous equation in x, y, z represents a cone with vertex at the origin.
3. Find the equation of a cone with vertex at $V(\alpha, \beta, \gamma)$
4. Find the condition that the general equation of the second degree should represent a cone.
5. Find the equation of the right circular cone vertex at $(\alpha, \beta, \gamma)$, semi vertical angle $\theta$, direction ratios of the axis are a, b, c.
6. Find the equation of the right circular cone satisfying the following (i) Vertex at origin, semi vertical angle $\theta$, direction ratios of the axis a, b, c. (ii) Vertex at $(\alpha, \beta, \gamma)$, semi vertical angle $\theta$, direction cosine of the axis l, m, n. (iii) Vertex at origin, semi vertical angle $\theta$, direction cosine of the axis l, m, n.
7. Find the equation of a right circular cone satisfying the following (i) vertex at the origin, z-axis as the axis, semi vertical angle $\theta$ (ii) vertex at the origin, X-axis as the axis, semi vertical angle $\theta$ (iii) vertex at the origin, Y-axis as the axis, semi vertical angle $\theta$
8. Find the equation of the tangent plane to the cone at P $(x_1, y_1, z_1)$.
9. Find the equation of cylinder when guiding curve is on XY plane whose generators are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$
10. Find the equation of cylinder whose generators intersect the guiding curve & parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$

11. Find the equation of right circular cylinder whose axis is the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ & whose radius is r.
12. Find the point of intersection of the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ & the conicoid $Ax^2 + By^2 + Cz^2 = 1$
13. Find the condition that the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ may be tangent to the conicoid $Ax^2 + By^2 + Cz^2 = 1$
14. Find the condition that the line lx + my + nz = p is tangent to the conicoid $Ax^2 + By^2 + Cz^2 = 1$.
15. Find the equation of the tangent plane to the conicoid $Ax^2 + By^2 + Cz^2 = 1$ at a point P $(x_1, y_1, z_1)$ on it.
16. Find the equation of normal to the conicoid $Ax^2 + By^2 + Cz^2 = 1$ at the point P $(x_1, y_1, z_1)$ on it.

**Q.3 Examples** **( 4 marks each )**

1. Find the equation of cone whose vertex is at the origin & which passes through the curve given by the equation $ax^2 + by^2 + cz^2 = 1$ & lx + my + nz = p.
2. Prove that the equation of the cone whose vertex is the origin & base the curve z = k, f(x, y) = 0 is $f\left(\frac{xk}{z}, \frac{yk}{z}\right) = 0$.
3. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the co-ordinate axis in points A, B, & C. Prove that the equation of the cone generated by the lines drawn from the origin to meet the circle ABC is $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$
4. If O is the origin, find the equation of the cone generated by the line OP as the point P describes the curve whose equation are $x^2 + y^2 + z^2 + x - 2y + 3z - 4 = 0$, $x^2 + y^2 + z^2 + 2x - 3y + 4z - 5 = 0$.
5. Obtain the general equation of the cone which passes through the three axes.
6. Obtain the equation of the cone which passes through the axes & the lines $\frac{x}{2} = \frac{y}{1} = \frac{z}{3}$ & $\frac{x}{-3} = \frac{y}{1} = \frac{z}{-2}$.
7. Obtain the equation of the cone which passes through the axes & the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ & $\frac{x}{3} = \frac{y}{1} = \frac{z}{-4}$.
8. Obtain the equation of the cone which passes through the axes & the lines $\frac{x}{3} = \frac{y}{-2} = \frac{z}{1}$ & $\frac{x}{1} = \frac{y}{-1} = \frac{z}{3}$.
9. Find the equation of the cone with vertex at the origin & containing the curve $x^2 + y^2 = 4$ & z = 5.
10. Examine whether the following equation represents a cone $5x^2 + 3y^2 + z^2 - 2xy - 6yz - 4xz + 6y + 8y + 10z - 26 = 0$, if it represents a cone, find its vertex.
11. Find the equation of right circular cone with its vertex at (1, -2, -1) semi vertical angle $60^0$ & axis $\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z+1}{5}$.

12. Find the equation of right circular cone passes through the point (1, 1, 2) has its axis the line $6x = 3y = 4z$ & vertex at the origin.
13. Find the equation of the cone with its vertex at the origin & passing through the curve $x^2 + y^2 + z^2 – 2x + 2y + 4z – 3 = 0$ & $x^2 + y^2 + z^2 + 2x + 4y + 6z – 11 = 0$.
14. Find the enveloping cone of the sphere $x^2 + y^2 + z^2 – 2x + 4z – 1 = 0$ with its vertex at (1, 1, 1).
15. Verify that the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{3}$ is the generator of the cone $x^2 + y^2 + z^2 + 4xy – xz = 0$
16. The line $3x + 2y – z = 0$ , $x + 3y + 2z = 0$ is a generator of the cone $2x^2 + y^2 - z^2 + 3yz – 2xz + axy = 0$, find the value of a.
17. Prove that the equation $4x^2 – y^2 + 2z^2 + 2xy – 3yz + 12x – 11y + 6z + 4 = 0$ represents a cone whose vertex is (-1, -2, -3).
18. Prove that the equation $x^2 – 2y^2 + 3z^2 – 4xy + 5yz – 6xz + 8x – 19y – 2z – 20 = 0$ represents a cone & find the vertex.
19. Show that the equation $x^2 – 2y^2 + 4z^2 + 4xy + 6yz – 2zx + 6x – 30y + 14z = 0$ represents a quadratic cone & find its vertex.
20. Examine whether the following equation represents a cone $4x^2+ 3y^2 - 5z^2 - 6yz – 8x + 16z – 4 = 0$ if it represents a cone find its vertex.
21. Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$.
22. Obtain the equation of a right circular cone which passes through the point (2, -1, -1) & has vertex at (4, 3, -2) & whose axis is parallel to the line $\frac{x}{1} = \frac{y}{1} = \frac{z}{2}$.
23. Find the equation of right circular cone whose vertex is at origin & axis is along $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ & which has a semi-vertical angle of $30^0$.
24. Find the equation of right circular cone whose vertex is at (2, -1, 4) & axis is along $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$ & semi-vertical angle is$\cos^{-1}\frac{4}{\sqrt{6}}$.
25. Find the equation of cone with vertex is at V(1, 2, -3) , axis is along $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z+3}{-2}$ & semi-vertical angle is $\cos^{-1}\frac{1}{\sqrt{3}}$
26. Obtain the equation of the right circular cone which passes through the point (1, 1, -1) & has the vertex at (-1, 3, -2) & whose axis is parallel to the line $x = y = z$.
27. Obtain the equation of the right circular cone which passes through the point Q(2, 1, 3) & has the vertex at V(1, 1, 2) & axis is parallel to the line $\frac{x-2}{2} = \frac{y-1}{-4} = \frac{z+2}{3}$
28. Find the equation of right circular cone whose vertex is (2, -3, 5) whose axis makes equal angles with the axes of co-ordinates & whose vertical angle is `of $60^0$.
29. Find the equation of right circular cone whose vertex is origin, axis is Z-axis & semi vertical angle is of $30^0$.
30. Find the equation of cone whose vertex is at origin & generators touching the sphere $x^2 + y^2 + z^2 - 2x + 4z = 1$.
31. Find the equation of the cylinder whose generators are parallel to the axis of Z & intersect the curve $ax^2 + by^2 + cz^2 = 1$ , $lx + my + nz = p$.

32. Show that the lines drawn through the points of the circle x + y + z – 1 = 0 $x^2 + y^2 + z^2 - 4 = 0$ parallel to the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$, generates the cylinder.
33. Obtain the equation of the cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ & whose guiding curve is $x^2 + 2y^2 = 1$, z = 3
34. Find the equation of cylinder with generators parallel to the line $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ & with generator parallel to guiding curves, $x^2 + 2y^2 + 6xy - 2z + 8 = 0$ , x - 2y + 3 = 0.
35. The axis of a right circular cylinder of the radius 2 is $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{2}$ find its equation.
36. Find the equation of right circular cylinder whose axis is $\frac{x-2}{2} = \frac{y-1}{1} = \frac{z}{3}$ & which passes through the point (0, 0, 3).
37. Find the equation of right circular cylinder whose radius 2, whose axis passes through the point (1, 2, 3) & has direction cosine proportional to 2, -3, 6.
38. Find the equation of right circular cylinder of radius 3 & having axis the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{5-z}{1}$.
39. Find the equation of right circular cylinder described on the base circle through (1, 0, 0), (0, 1, 0), (0, 0, 1).
40. Find the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having its generator parallel to x = y = z , also find its guiding curve.
41. Find the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ having its generators parallel to the line x = -2y = 2z.
42. Find the equation of the right circular cylinder through the three points A (a, 0, 0), B (0, a, 0) & C (0, 0, a) as the guiding circle.
43. Show that the line $\frac{x-3}{2} = \frac{y-3}{3} = \frac{z+3}{-4}$ touches the ellipsoid $\frac{x^2}{8} + \frac{y^2}{9} + \frac{z^2}{4} = 1,$ find the point of contact & the tangent plane containing the given tangent line.
44. Find the equation of the tangent plane at the point P (-2, 2, 3) to the ellipsoid $4x^2 + y^2 + 5z^2 = 65$, find the equation of normal at P.
45. Show that the plane 7x + 5y + 3z = 30 touches the ellipsoid $7x^2 + 5y^2 + 3z^2 = 60$ ,& find the point of contact.
46. Show that the plane 6x – 5y – 6z = 20 touches the hyperboloid $4x^2 - 5y^2 + 6z^2 = 40$ ,& find the point of contact.
47. Prove that the plane 3x + 12y – 6z – 17 = 0 touches the conicoid $3x^2 - 6y^2 + 9z^2 + 17 = 0$. Also find point of contact.
48. Find the equation of tangent planes to the conicoid $2x^2 - 6y^2 + 3z^2 = 5$ which passes through the line x + 9y – 3z = 0, 3x – 3y + 6z – 5 = 0.
49. Find the equation of tangent planes to the conicoid $7x^2 + 5y^2 + 3z^2 = 60$ which passes through the line 7x + 10y – 30 = 0, 5y - 3z = 0.
50. Prove that the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z-2}{-1}$ touches the hyperboloid $4x^2 - 5y^2 - 6z^2 + 35 = 0$. Find the point of contact.
51. Find the point of intersection of the line $\frac{x-4}{3} = \frac{y-1}{-1} = \frac{z-2}{3}$ & the ellipsoid $2x^2 + 3y^2 + 7z^2 = 21$.

52. Find the point of intersection of the line $\frac{x-9}{3} = \frac{y-4}{1} = \frac{z+3}{3}$ & the hyperboloid $4x^2 - 3y^2 + z^2 = 33$.
53. Find the equation of tangent plane & normal at point (1, 2, 4) to the hyperboloid $7x^2 - 3y^2 - z^2 + 21 = 0$.
54. Find the equation of tangent planes to the conicoid $4x^2 - 5y^2 + 7z^2 + 13 = 0$ which are parallel to the plane $4x + 20y - 21z = 0$. Also find points of contact.
55. Show that the line $\frac{x-10}{8} = \frac{y+6}{-9} = \frac{z-16}{-14}$ is a normal to the conicoid $4x^2 - 3y^2 + 7z^2 = 17$, Find the foot of the normal.
56. Show that the line $\frac{x+6}{7} = \frac{y-8}{-6} = \frac{z-8}{-6}$ is a normal to the conicoid $7x^2 - 3y^2 + z^2 + 21 = 0$; Find the foot of the normal.

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