NATIONAL INSTITUTE OF TECHNOLOGY, CALICUT
Seventh Semester B.Tech End Semester Examination- November 2007

EC 455T Wavelets

Time: 3 hrs.   **Marks: 50**

Answer All Questions

I   Check whether the following functions are admissible wavelets?

         1. $(1-2t^2)e^{-t^2}$

         2. $e^{-t^2} \, Cos\left(\pi\sqrt{\dfrac{2}{\ell n2}}\,t\right)$ [marks3]

II   Given $F(t) = \left\{ \begin{cases} 0 & t < 0 \\ t & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases} \right\}$

     Show that the wavelet transform of F(t) using Haar Wavelet is

$$W(a,b) = sgn(a)\,\frac{2F(b+\frac{a}{2}) - F(b) - F(b+a)}{\sqrt{|a|}}$$ [marks3]

III a)   Design a Filter bank to compute the 4 point DFT of a Digital Sequence. Obtain DFT of the following sequence using the above filter bank. $x(n) = \{ 4, 8, 4, 10 \}$ [marks3]

    b)   Obtain 1 level DWT of the above sequence using Haar Wavelet implemented with Mallat Algorithm.   [marks1]

    c)   Compare the performance of the two transforms in terms of energy compaction.   [marks1]

IV   Design Daubechies Orthogonal Wavelet system with two vanishing moments using time domain approach.   [marks4]

V   Derive the Mallat Filterbank structure (Analysis & Synthesis) for a Biorthogonal Wavelet System starting from the basic two scale equations.   [marks4]

     How will you get time information from Mallat's structure?   [marks1]

VI   What are the advantages of Wavelet Packet Transform compared to Wavelet Transform? How will you determine the best wavelet packet bases for a given data?   [marks3]

VII   Distinguish between Time Domain Masking and Frequency Domain Masking in the context of hearing process , with necessary diagrams.   [marks3]

VIII   Show that if $\displaystyle\int_{\infty}^{+\infty} t^k \psi(t)dt = 0$ then

$$\left| \frac{d^k}{d\omega^k} \psi(\omega) \right|_{\omega=0} = 0 \qquad \&$$

$$\frac{d^k}{d\omega^k} \left[H^*(\omega + \pi) \right]_{\omega=\pi} = 0$$ [marks3]

IX   For the seven-level decomposition shown below,

21	6	15	12
-6	3	6	3
3	-3	0	-3
3	0	0	0

a)   Find the bit stream or labels generated by the Embedded Zerotree Wavelet (EZW) coder, after three steps of significant and refinement passes. Also, determine the list of significant coefficients.   [marks6]

b)   Find the Mean Squared Error between the original DWT coefficients and reconstructed coefficients after three passes.   [marks2]

X   Write short notes on :

   a) Regularity of a wavelet

   b) Vanishing Moments of a wavelet

   c) Admissibility condition

   d) Multi-resolution property of wavelet systems   [marks4]

XI a)   Obtain two level DWT of the following sequence using Haar Wavelet Transform with lifting scheme.   [marks4]

                10, 20, 25, 15, 20, 15, 10, 15

b)   Compare the computational complexities of Mallat algorithm & Lifting scheme for this case in terms of total number of multiplications and additions.   [marks1]

c)   Compute the energy stored in the four low frequency sub-bands.   [marks1]

d)   Threshold few coefficients and perform inverse DWT using Lifting scheme such that the reconstructed sequence retains approximately 90% of the total energy of the original sequence.   [marks3]