NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

DEPARTMENT OF MATHEMATICS

S2 B.TECH END SEMESTER EXAMINATION (WINTER SEMESTER) APRIL 2011

MA1002 MATHEMATICS II

Max.marks : 50 **Time: 3 Hours**

1. Find the rate of change of $\phi = 4x^2 + y^2 - 16z$ at (2,4,2) in the direction of a normal to the plane $x+2y+2z = 9$. What is the maximum rate of change of ϕ at (2,4,2)? (3)

2. Prove that $\nabla r^n = n\, r^{n-2}\,\overline{r}$ where $\overline{r} = xi + yj + zk$ and $r = (x^2+y^2+z^2)^{1/2}$. (3)

3. Find the work done when a force $\overline{F} = (x^2 - y^2 + x)i - (2xy + y)j$ moves a particle in the XY plane from (0,0) to (1,1) along the parabola $y^2 = x$. Is the work done different when the path is a straight line $y = x$? Justify your answer. (3)

4. Verify Green's theorem for $\int_C (xy^2 - 2xy)dx + (x^2y + 3)dy$ around the curve C bounded by $y^2 = 8x$ and $x=2$ in the XY Plane. (4)

5. i) Prove that every gradient field $\overline{F}$, of a scalar function ϕ has Curl $\overline{F} = 0$.

ii) Is there a vector field $\overline{G}$ for which Curl $\overline{G} = xi + yj + zk$? Give reason. (3)

6. Verify Stoke's Theorem for $\overline{F} = (y - z + 4)i + (yz+3)j - xz\,k$ and S is an open surface of the cube formed by the planes x=0, x=3, y=0, y=3, and z=3 above the XY plane. (4)

7. If the eigen values of a square matrix are -1, 2 and 5 find the eigen values of $(5A + A^2)^{-1}$ (2)

8. Obtain a basis and the dimension of the solution space of the homogenous system of equations

$2x_1 + x_2 - x_3 + x_5 = 0$

$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$

$x_1 + x_2 - 2x_3 - x_5 = 0$

$x_3 + x_4 + x_5 = 0.$ (3)

9. If $\overline{u}, \overline{v}$ and $\overline{w}$ are linearly independent vectors show that $\overline{u} + \overline{v}$, $\overline{v} + \overline{w}$ and $\overline{w} + \overline{u}$ are also linearly independent. (3)

P.T.O

10. Find k such that the linear transformation $T : R^3 \longrightarrow R^3$ defined by $T(x,y,z) = (kx + y + 2z,\ x - y - 2z,\ x + y + 4z)$ is of Nullity Zero. For k =1 is T invertible? If so, find T^{-1}. **(4)**

11. Using Cayley Hamilton Theorem find the inverse of $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{bmatrix}$ **(2)**

12. Let $T : R^3 \longrightarrow R^3$ defined by $T(a,b,c) = (a - b + c,\ 2a + b - c,\ -a - 2b + 2c)$ be a linear transformation. Obtain bases for the Range and Null space and hence verify Rank Nullity Theorem. **(4)**

13. i) Define a real inner product space.
ii) Let $\bar{u} = (u_1,u_2)$ and $\bar{v} = (v_1,v_2)\ \varepsilon\ R^2$ and define $\langle \bar{u}, \bar{v} \rangle = 2\, u_1v_1 + u_1v_2 + u_2v_1 + 2\, u_2v_2$. Is this function an inner product on R^2 ? Give reason. **(3)**

14. Using Gram Schmidt Orthogonalization process find an orthonormal basis for the space spanned by (1,1,0), (1,0,1), (0,1,1) using dot product as the inner product. **(3)**

15. Define Hermitian, Unitary and Orthogonal matrix. Show that eigen values of a skew hermitian matrix are either zero or purely imaginary. **(3)**

16. Find the rank, index and signature of the Quadratic form $Q = 10\, x_1^2 + 2\, x_2^2 + 15\, x_3^2 + 6x_2x_3 - 10x_1x_2 - 4x_1x_3$. **(3)**
