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EXAMINATION

12 September 2006 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

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1 (a) Distinguish between a future and an option.
(b) Explain why convertibles have "option-like" characteristics.

[3]

2 An individual makes an investment of £4m per annum in the first year, £6m per annum in the second year and £8m per annum in the third year. The investments are made continuously throughout each year. Calculate the accumulated value of the investments at the end of the third year at a rate of interest of 4% per annum effective.

[3]

3 An individual has invested a sum of £10m. Exactly one year later, the investment is worth £11.1m. An index of prices has a value of 112 at the beginning of the investment and 120 at the end of the investment. The investor pays tax at 40% on all money returns from investment. Calculate:

(a) The money rate of return per annum before tax.
(b) The rate of inflation.
(c) The real rate of return per annum after tax.

[4]

4 An investor is able to purchase or sell two specially designed risk-free securities, A and B. Short sales of both securities are possible. Security A has a market price of 20p. In the event that a particular stock market index goes up over the next year, it will pay 25p and, in the event that the stock market index goes down, it will pay 15p. Security B has a market price of 15p. In the event that the stock market index goes up over the next year, it will pay 20p and, in the event that the stock market index goes down, it will pay 12p.

(i) Explain what is meant by the assumption of "no arbitrage" used in the pricing of derivative contracts. [2]

(ii) Find the market price of B, such that there are no arbitrage opportunities and assuming the price of A remains fixed. Explain your reasoning. [2]

[Total 4]

5 (i) Calculate the time in days for £3,600 to accumulate to £4,000 at:

(a) a simple rate of interest of 6% per annum
(b) a compound rate of interest of 6% per annum convertible quarterly
(c) a compound rate of interest of 6% per annum convertible monthly

[4]

(ii) Explain why the amount takes longest to accumulate in (i)(a) [1]

[Total 5]

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6 The rate of interest is a random variable that is distributed with mean 0.07 and variance 0.016 in each of the next 10 years. The value taken by the rate of interest in any one year is independent of its value in any other year. Deriving all necessary formulae calculate:

(i) The expected accumulation at the end of ten years, if one unit is invested at the beginning of ten years. [3]

(ii) The variance of the accumulation at the end of ten years, if one unit is invested at the beginning of ten years. [5]

(iii) Explain how your answers in (i) and (ii) would differ if 1,000 units had been invested. [1]

[Total 9]

7 A life insurance fund had assets totalling £600m on 1 January 2003. It received net income of £40m on 1 January 2004 and £100m on 1 July 2004. The value of the fund was:

£450m on 31 December 2003;
£500m on 30 June 2004;
£800m on 31 December 2004.

(i) Calculate, for the period 1 January 2003 to 31 December 2004, to three decimal places:

(a) The time weighted rate of return per annum.

(b) The linked internal rate of return, using sub intervals of a calendar year.

[8]

(ii) Explain why the linked internal rate of return is higher than the time weighted rate of return. [2]

[Total 10]

8 The force of interest $\delta(t)$ at time t is $at + bt^2$ where a and b are constants. An amount of £100 invested at time $t = 0$ accumulates to £150 at time $t = 5$ and £230 at time $t = 10$.

(i) Calculate the values of a and b. [5]

(ii) Calculate the constant force of interest that would give rise to the same accumulation from time $t = 0$ to time $t = 10$. [2]

(iii) At the force of interest calculated in (ii), calculate the present value of a continuous payment stream of $20e^{0.05t}$ paid between from time $t = 0$ to time $t = 10$. [4]

[Total 11]

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9 An individual took out a loan of £100,000 to purchase a house on 1 January 1980. The loan is due to be repaid on 1 January 2010 but the borrower can repay the loan early if he wishes. The borrower pays interest on the loan at a rate of 6% per annum convertible monthly, paid in arrears. The loan instalments only cover the interest on the loan. At the same time, the borrower took out a thirty-year investment policy, which was expected to repay the loan, and into which monthly premiums were paid, in advance, at a rate of £1,060 per annum. The individual was told that premiums in the investment policy were expected to earn a rate of return of 7% per annum effective. After twenty years, the individual was informed that the premiums had only earned a rate of return of 4% per annum effective and that they would continue to do so for the final ten years of the policy. The borrower agrees to increase his monthly payments into the investment policy to £5,000 per annum for the final ten years.

(a) Calculate the amount to which the investment policy was expected to accumulate at the time it was taken out.

(b) Calculate the amount by which the investment policy would have fallen short of repaying the loan had extra premiums not been paid for the final ten years.

(c) Calculate the amount of money the individual will have, after using the proceeds of the investment policy to repay the loan, after allowing for the increase in premiums.

(d) Suggest another course of action the borrower could have taken which would have been of higher value to him, explaining why this higher value arises.

(e) Calculate the level annual instalment that the investor would have had to pay from outset if he had repaid the loan in equal instalments of interest and capital.

[11]

10 A financial regulator has brought in a new set of regulations and wishes to assess the cost of them. It intends to conduct an analysis of the costs and benefits of the new regulations in their first twenty years.

The costs are estimated to be as follows:

- The cost to companies who will need to devise new policy terms and computer systems is expected to be incurred at a rate of £50m in the first year increasing by 3% per annum over the twenty year period.

- The cost to financial advisers who will have to set up new computer systems and spend more time filling in paperwork is expected to be incurred at a rate of £60m in the first year, £19m in the second year, £18m in the third year, reducing by £1m every year until the last year, when the cost incurred will be at a rate of £1m.

- The cost to consumers who will have to spend more time filling in paperwork and talking to their financial advisers is expected to be incurred at a rate of £10m in the first year, increasing by 3% per annum over the twenty year period.

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The benefits are estimated as follows:

- The benefit to consumers who are less likely to buy inappropriate policies is estimated to be received at a rate of £30m in the first year, £33m in the second year, £36m in the third year and so on, rising by £3m per year until the end of twenty years.

- The benefit to companies who will spend less time dealing with complaints from customers is estimated to be received at a rate of £12m per annum for twenty years.

Calculate the net present value of the benefit or cost of the regulations in their first twenty years at a rate of interest of 4% per annum effective. Assume that all costs and benefits occur continuously throughout the year.

[12]

11 (i) Describe the characteristics of an index-linked government bond. [3]

(ii) On 1 July 2002, the government of a country issued an index-linked bond of term seven years. Coupons are paid half-yearly in arrears on 1 January and 1 July each year. The annual nominal coupon is 2%. Interest and capital payments are indexed by reference to the value of an inflation index with a time lag of eight months.

You are given the following values of the inflation index.

Date	*Inflation index*
November 2001	110.0
May 2002	112.3
November 2002	113.2
May 2003	113.8

The inflation index is assumed to increase continuously at the rate of 2½% per annum effective from its value in May 2003.

An investor, paying tax at the rate of 20% on coupons only, purchased the stock on 1 July 2003, just after a coupon payment had been made.

Calculate the price to this investor such that a real net yield of 3% per annum convertible half yearly is obtained and assuming that the investor holds the bond to maturity. [10]

[Total 13]

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12 A pension fund has the following liabilities: annuity payments of £160,000 per annum to be paid annually in arrears for the next 15 years and a lump sum of £200,000 to be paid in ten years. It wishes to invest in two fixed-interest securities in order to immunise its liabilities. Security A has a coupon rate of 8% per annum and a term to redemption of eight years. Security B has a coupon rate of 3% per annum and a term to redemption of 25 years. Both securities are redeemable at par and pay coupons annually in arrear.

(i) Calculate the present value of the liabilities at a rate of interest of 7% per annum effective. [2]

(ii) Calculate the discounted mean term of the liabilities at a rate of interest of 7% per annum effective. [4]

(iii) Calculate the nominal amount of each security that should be purchased so that both the present value and discounted mean terms of assets and liabilities are equal. [7]

(iv) Without further calculation, comment on whether, if the conditions in (iii) are fulfilled, the pension fund is likely to be immunised against small, uniform changes in the rate of interest. [2]

[Total 15]

END OF PAPER

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