Faculty of Actuaries **Institute of Actuaries**

EXAMINATION

6 September 2006 (am)

Subject CT4 (104) — Models (104 Part) Core Technical

Time allowed: One and a half hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 6 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

B1 Calculate ${}_{0.25}p_{80}$ and ${}_{0.25}p_{80.5}$, using the ELT15 (Females) mortality table and assuming a uniform distribution of deaths. [4]

B2 A national mortality investigation is carried out over the calendar years 2002, 2003 and 2004. Data are collected from a number of insurance companies.

Deaths during the period of the investigation, θ_x, are classified by age nearest at death.

Each insurance company provides details of the number of in-force policies on 1 January 2002, 2003, 2004 and 2005, where policyholders are classified by age nearest birthday, $P_x(t)$.

(i) (a) State the rate year implied by the classification of deaths.
(b) State the ages of the lives at the start of the rate interval.

[1]

(ii) Derive an expression for the exposed to risk, in terms of $P_x(t)$, which may be used to estimate the force of mortality in year t at each age. State any assumptions you make. [3]

(iii) Describe how your answer to (ii) would change if the census information provided by some companies was $P_x^*(t)$, the number of in-force policies on 1 January each year, where policyholders are classified by age last birthday.

[3]
[Total 7]

B3 An investigation was undertaken into the effect of a new treatment on the survival times of cancer patients. Two groups of patients were identified. One group was given the new treatment and the other an existing treatment.

The following model was considered:

$$h_i(t) = h_0(t) \cdot \exp\left(\underline{\beta}^T \underline{z}\right)$$

where: $h_i(t)$ is the hazard at time t, where t is the time since the start of treatment

$h_0(t)$ is the baseline hazard at time t

$\underline{z}$ is a vector of covariates such that:

z_1 = sex (a categorical variable with 0 = female, 1 = male)

z_2 = treatment (a categorical variable with 0 = existing treatment, 1 = new treatment)

and $\underline{\beta}$ is a vector of parameters, (β_1, β_2).

The results of the investigation showed that, if the model is correct:

A the risk of death for a male patient is 1.02 times that of a female patient; and

B the risk of death for a patient given the existing treatment is 1.05 times that for a patient given the new treatment

(i) Estimate the value of the parameters β_1 and β_2. [3]

(ii) Estimate the ratio by which the risk of death for a male patient who has been given the new treatment is greater or less than that for a female patient given the existing treatment. [2]

(iii) Determine, in terms of the baseline hazard only, the probability that a male patient will die within 3 years of receiving the new treatment. [2]
[Total 7]

B4 An investigation took place into the mortality of persons between exact ages 60 and 61 years. The table below gives an extract from the results. For each person it gives the age at which they were first observed, the age at which they ceased to be observed and the reason for their departure from observation.

Person	*Age at entry*		*Age at exit*		*Reason for exit*
	years	*months*	*years*	*months*	
1	60	0	60	6	withdrew
2	60	1	61	0	survived to 61
3	60	1	60	3	died
4	60	2	61	0	survived to 61
5	60	3	60	9	died
6	60	4	61	0	survived to 61
7	60	5	60	11	died
8	60	7	61	0	survived to 61
9	60	8	60	10	died
10	60	9	61	0	survived to 61

(i) Estimate q_{60} using the Binomial model. [5]

(ii) List the strengths and weaknesses of the Binomial model for the estimation of empirical mortality rates, compared with the Poisson and two-state models.
[3]
[Total 8]

B5 A life insurance company has carried out a mortality investigation. It followed a sample of independent policyholders aged between 50 and 55 years. Policyholders were followed from their 50th birthday until they died, they withdrew from the investigation while still alive, or they celebrated their 55th birthday (whichever of these events occurred first).

(i) Describe the censoring that is present in this investigation. [2]

An extract from the data for 12 policyholders is shown in the table below.

Policyholder	*Last age at which policyholder was observed (years and months)*	*Outcome*
1	50 years 3 months	Died
2	50 years 6 months	Withdrew
3	51 years 0 months	Died
4	51 years 0 months	Withdrew
5	52 years 3 months	Withdrew
6	52 years 9 months	Died
7	53 years 0 months	Withdrew
8	53 years 6 months	Withdrew
9	54 years 3 months	Withdrew
10	54 years 3 months	Died
11	55 years 0 months	Still alive
12	55 years 0 months	Still alive

(ii) Calculate the Nelson-Aalen estimate of the survival function. [5]

(iii) Sketch on a suitably labelled graph the Nelson-Aalen estimate of the survival function. [2]

[Total 9]

B6 (i) (a) Describe the general form of the polynomial formula used to graduate the most recent standard tables produced for use by UK life insurance companies.

(b) Show how the Gompertz and Makeham formulae arise as special cases of this formula.

[3]

(ii) An investigation was undertaken of the mortality of persons aged between 40 and 75 years who are known to be suffering from a degenerative disease. It is suggested that the crude estimates be graduated using the formula:

$$\overset{o}{\mu}_{x+\frac{1}{2}} = \exp\left[b_0 + b_1\left(x+\frac{1}{2}\right) + b_2\left(x+\frac{1}{2}\right)^2\right].$$

(a) Explain why this might be a sensible formula to choose for this class of lives.

(b) Suggest two techniques which can be used to perform the graduation.

[3]

(iii) The table below shows the crude and graduated mortality rates for part of the relevant age range, together with the exposed to risk at each age and the standardised deviation at each age.

Age last birthday	*Graduated force of mortality*	*Crude force of mortality*	*Exposed to risk*	*Standardised deviation*
x	$\overset{o}{\mu}_{x+1/2}$	$\hat{\mu}_{x+1/2}$	E_x^c	$z_x = \dfrac{E_x^c\left(\hat{\mu}_{x+1/2} - \overset{o}{\mu}_{x+1/2}\right)}{\sqrt{E_x^c\,\overset{o}{\mu}_{x+1/2}}}$
50	0.08127	0.07941	340	-0.12031
51	0.08770	0.08438	320	-0.20055
52	0.09439	0.09000	300	-0.24749
53	0.10133	0.10345	290	0.11341
54	0.10853	0.09200	250	-0.79336
55	0.11600	0.10000	200	-0.66436
56	0.12373	0.11176	170	-0.44369
57	0.13175	0.12222	180	-0.35225

Test this graduation for:

(a) overall goodness-of-fit

(b) bias; and

(c) the existence of individual ages at which the graduated rates depart to a substantial degree from the observed rates

[9]
[Total 15]

END OF PAPER