EXAMINATION

5 September 2006 (am)

Subject CT6 — Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 The loss function under a decision problem is given by:

	θ_1	θ_2	θ_3
D1	11	9	19
D2	10	13	17
D3	7	13	10
D4	16	5	13

(i) State which decision can be discounted immediately and why. [2]

(ii) Explain what is meant by the minimax criterion and determine the minimax solution in this case. [2]
[Total 4]

2 A sequence of pseudo-random numbers from a uniform distribution over the interval [0, 1] has been generated by a computer.

(i) Explain the advantage of using pseudo-random numbers rather than generating a new set of random numbers each time. [1]

(ii) Use examples to explain how a sequence of pseudo-random numbers can be used to simulate observations from:

(a) a continuous distribution
(b) a discrete distribution [4]
[Total 5]

3 State the Markov property and explain briefly whether the following processes are Markov:

AR(4);
ARMA (1, 1).
[5]

4 An insurer insures a single building. The probability of a claim on a given day is p independently from day to day. Premiums of 1 unit are payable on a daily basis at the start of each day. The claim size is independent of the time of the claim and follows an exponential distribution with mean $1/\lambda$. The insurer has a surplus of U at time zero.

(i) Derive an expression for the probability that the first claim results in the ruin of the insurer. [6]

(ii) If $p = 0.01$ and $\lambda = 0.0125$ find how large U must be so that the probability that the first claim causes ruin is less than 1%. [2]
[Total 8]

5 (i) Let p be an unknown parameter, and let $f(p|\underline{x})$ denote the probability density of the posterior distribution of p given information $\underline{x}$. Show that under all-or-nothing loss the Bayes estimate of p is the mode of $f(p|\underline{x})$. [2]

(ii) Now suppose p is the proportion of the population carrying a particular genetic condition. Prior beliefs about p have a $U(0, 1)$ distribution. A sample of size N is taken from the population revealing that m individuals have the genetic condition.

(a) Suggest why the $U(0, 1)$ distribution has been chosen as the prior, and derive the posterior distribution of p.

(b) Calculate the Bayes estimate of p under all-or-nothing loss.

[6]
[Total 8]

6 The table below shows cumulative paid claims and premium income on a portfolio of general insurance policies.

Underwriting Year	*Development Year* 0	1	2	*Premium Income*
2002	38,419	77,112	91,013	120,417
2003	31,490	78,504		117,101
2004	43,947			135,490

(i) Assuming an ultimate loss ratio of 93% for underwriting years 2003 and 2004, calculate the Bornhuetter-Ferguson estimate of outstanding claims for this triangle. [8]

(ii) State the assumptions underlying this estimate. [2]
[Total 10]

7 The random variable W has a binomial distribution such that

$$P(W = w) = \binom{n}{w} \mu^w (1-\mu)^{n-w} \quad (w = 0, 1, 2, \ldots, n)$$

Let $Y = \dfrac{W}{n}$.

(i) Write down an expression for $P(Y = y)$, for $y = 0, \frac{1}{n}, \frac{2}{n}, \ldots, 1$. [1]

(ii) Express the distribution of Y as an exponential family and identify the natural parameter and the dispersion parameter. [3]

(iii) Derive an expression for the variance function. [3]

(iv) For a set of n independent observations of Y, derive an expression of the scaled deviance. [3]
[Total 10]

8 (i) Let X denote the claim amount under an insurance policy, and suppose that X has a probability density $f_X(x)$ for $x > 0$. The insurer has an individual excess of loss reinsurance arrangement with a retention of £M. Let Y be the amount paid by the insurer net of reinsurance. Express Y in terms of X and hence derive an expression for the probability density function of Y in terms of $f_X(x)$. [3]

For a particular class of policy X is believed to follow a Weibull distribution with probability density function

$$f_X(x) = 0.75cx^{-0.25} e^{-cx^{0.75}} \quad (x > 0)$$

where c is an unknown constant. The insurer has an individual excess of loss reinsurance arrangement with retention £500. The following claims data are observed:

Claims below retention:	78, 104, 116, 135, 189, 243, 270, 350, 411, 491
Claims above retention:	3 in total
Total number of claims:	13

(ii) Estimate c using maximum likelihood estimation. [7]

(iii) Apply the method of percentiles using the median claim to estimate c. [4]
[Total 14]

9 An insurer operates a No Claims Discount system with three levels of discount:

	Discount
Level 0	0%
Level 1	20%
Level 2	50%

The annual premium in level 0 is £650.

If a policyholder makes no claims in a policy year, they move to the next high discount level (or remain at level 2). In all other cases they move to (or remain at) discount level 0.

For a policyholder who has not yet had an accident in a policy year, the probability of an accident occurring is 0.1. The time at which an accident occurs in the policy year is denoted by T, where

$0 \le T \le 1$;
$T = 0$ means that the accident occurs at the start of the policy year;
$T = 1$ means that the accident occurs at the end of the policy year.

It is assumed that T has a uniform distribution.

Given that a policyholder has had their first accident, the probability of them having a second accident in the same policy year is $0.4(1 - T)$. It is assumed that a policyholder will not have more than two accidents in a policy year.

The cost of each accident has an exponential distribution with mean £1,000.

After each accident, the policyholder decides whether or not to make a claim by comparing the increase in the premium they would have to pay in the next policy year with the claim size. In doing this, they assume that they will have no further accidents.

(i) Show that the distribution of the number of accidents, K, that a policyholder has in a year is:

$P(K = 0) = 0.9$
$P(K = 1) = 0.08$
$P(K = 2) = 0.02$

[4]

(ii) For each level of discount, calculate the probability that a policyholder makes n claims in a policy year, where $n = 0, 1, 2$. [8]

(iii) Write down the transition matrix. [2]

(iv) Derive the steady state distribution. [3]
[Total 17]

10 (i) Let $I_k = \int_m^\infty x^k e^{-\beta x} dx$

where k is a non-negative integer.

Show that $I_0 = \frac{1}{\beta} e^{-\beta m}$

and $I_k = \frac{m^k}{\beta} e^{-\beta m} + \frac{k}{\beta} I_{k-1}$ $(k = 1, 2, 3, \ldots)$ [3]

For a certain portfolio of insurance policies the number of claims annually has a Poisson distribution with mean 25. Claim sizes have a gamma distribution with mean 100 and variance 5,000 and the insurer includes a loading of 10% in its premium.

The insurer is considering purchasing individual excess of loss reinsurance with retention m from a reinsurer that includes a loading of 15% in its premium.

Let X_I and X_R denote the amounts paid by the direct insurer and the reinsurer, respectively, on an individual claim.

(ii) Calculate the premium, c, charged by the direct insurer for this portfolio. [1]

(iii) Show that $E[X_R] = \frac{1}{50^2}(I_2 - mI_1)$ and hence that

$$E[X_R] = (m + 100)\, e^{-m/50}.$$ [7]

(iv) Use the result in (iii) to derive an expression for $E[X_I]$. [1]

(v) Derive an expression for the direct insurer's expected annual profit. [3]

(vi) The table below shows the direct insurer's expected annual profit (Profit) and probability of ruin (P(ruin)), for various values of the retention level, m:

m	*Profit*	*P(ruin)*
36	1.8	0.002
50	*	0.01
100	148.5	0.05

Calculate the missing value in the table and discuss the issues facing the direct insurer when deciding on the retention level to use. [4]

[Total 19]

END OF PAPER