

# INSTITUTE OF ACTUARIES OF INDIA

## EXAMINATIONS

29<sup>th</sup> October 2009

**Subject ST6 — Finance and Investment B**

**Time allowed: Three hours (14.45\* – 18.00 Hours)**

**Total Marks: 100**

### INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
2. *\* You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the answer sheet until instructed to do so by the supervisor*
4. *The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
5. *Attempt all questions, beginning your answer to each question on a separate sheet.*
6. *Mark allocations are shown in brackets.*

### AT THE END OF THE EXAMINATION

**Please return your answer book and this question paper to the supervisor separately.**

- Q 1)** An annual level perpetuity is a series of payment that pays equal amount of money every year forever
- a) If the yield curve is flat, prove that the duration of an annual level perpetuity is  $\frac{y}{1+y}$  where y is the yield per annum with annual compounding. (4)
- b) You are managing a portfolio of Rs. 100 million. Your target duration is 8 years, and you can choose from two bonds: a zero coupon bond with maturity of 6 years, and an annual level perpetuity, each currently yielding 10% per annum with annual compounding.
- i. How much of each bond will you hold in your portfolio?
- ii. How will these fractions change next year if target duration now is 7 years? (4)
- [8]**

- Q 2)** Consider a two year contract which pays out 95% the ratio of the terminal and initial value of the NSE Nifty or 110% if otherwise it would be less. That is, the payoff from a long position in the contract on the expiration date is given by:

$$X = \text{Max}(1.10, 0.95S_T^*)$$

where T is 2 years and  $S_T^* = \frac{S_T}{S_0}$

$S_T$  is the value of the Nifty on expiration of the contract and  $S_0$  is the current value of the Nifty. Value of Nifty follows geometric Brownian motion with an expected rate of return of 14% per annum (with continuous compounding) and a volatility of 35% per annum (with continuous compounding). The risk-free interest rate is 6% per annum (with continuous compounding) and the dividend yield on Nifty is 3% per annum (with continuous compounding).

- a) Determine today's price of contract X. (6)
- b) Explain the impact on the price computed in part a, if you ignore the dividends? Compute the approximate impact without recomputing the value of the option. (3)
- c) A firm wants something which is cheaper than the contract in (a). It is then suggested by you that the contract should be modified. You suggest that it should be a two year contract that pays out 95% times the ratio of the terminal and initial value of the Nifty or 110% if otherwise it would be less or 1.50 if otherwise it would be more. The modified payoff Y on expiration date is given by:

$$Y = \text{Min}\{\text{Max}(1.10, 0.95S_T^*), 1.50\}$$

Determine today's price of the modified contract. By what percentage the modified contract (Y) cheaper than the original contract (X)? (5)

**[14]**

- Q 3)** A forward start call option is a contract in which the holder receives at time  $T_1$ , a European call option with expiration date  $T_2$  ( $T_1 < T_2$ ) and strike price  $S_{T_1}$  (where  $S_{T_1}$  is the spot price of the underlying asset at time  $T_1$ ).
- a) Write down the terminal payoff at time  $T_2$  of a forward start call option. (1)
- b) Derive the arbitrage free price at time 0 of a forward call option either by solving the partial differential equations or by taking expectation under risk neutral measure. (4)
- [5]**

- Q 4)** Assume that the stock price follows the following process under the martingale measure  $Q$ .

$$dS_t = S_t \sigma d\bar{W}_t$$

$S_0$  is the initial value of stock

$\sigma$  is a constant

$\bar{W}_t$  denotes a  $Q$ -wiener process

Assume that the risk-free rate of interest is zero

- a) Prove that at  $t = 0$ , the price of a European call option on the stock with exercise price  $K$  and expiration date  $T$  is given by

$$(S_0 - K) \Phi\left(\frac{S_0 - K}{\sigma S_0 \sqrt{T}}\right) + \sigma S_0 \sqrt{T} \phi\left(\frac{S_0 - K}{\sigma S_0 \sqrt{T}}\right) \quad (4)$$

- b) Compute the delta of a call option given that it is priced using the formula in (a), and explain how it can be used for hedging if you have sold the option and intend to use the underlying itself for hedging. (4)
- [8]**

- Q 5)**
- a) A stock price is currently Rs.1000. Over each of the next two 6-months period it is expected to go up by 20% or down by 20%. The risk-free interest rate is zero.
- i) What is the value of a European call option with a strike price of Rs. 1000?
- ii) Find the replicating portfolio for the call option in (a) and verify that the portfolio is self-financing. (5)
- b) A stock price is currently Rs. 100. It is known that at the end of one year it will be either Rs. 150 or Rs. 50.
- i) What are the arbitrage bounds for the risk-free interest rate?
- ii) Given that the current price of a European call option with strike price of Rs. 108 and exercise time 1 year has been computed to Rs. 22. What is the interest rate with annual compounding? (3)
- c) A stock price is currently Rs. 500. It is known that at the end of one year it will be either Rs. 750 or Rs. 500 or Rs. 250. The risk-free rate of interest is 10% per annum with annual compounding. Under what conditions will the stock price be free from arbitrage? (2)
- [10]**

**Q 6)** The book value of a company's issued equity share capital is Rs. 500 million and the volatility of its equity is 30% per annum (with continuous compounding). The only debt that it has is Rs. 900 million and will have to be paid in seven years. The risk-free rate of interest is 5% (with continuous compounding). The company has 20 million shares outstanding.

- (i) Assuming that the debt is repaid directly from the company's fund at that time, state the share price at time 7 if the total value of the company at that time is: (a) Rs. 1200 million, (b) Rs. 350 million.
- (ii) Explain the Merton model and use it to derive an equation for the market value of shares in the company at time 0.

[5]

**Q 7)** Consider the Vasicek model

$$dr(t) = [\theta - \alpha r(t)]dt + \sigma d\bar{W}(t)$$

$$r(0) = r_0$$

where  $\alpha, \theta, \sigma$ , and  $r_0$  are constants.

- a) Derive an equation expressing  $r(t)$  in terms of  $r_0$ . [Hint: Use  $e^{\alpha t}$  as multiplier] (4)
- b) Determine the distribution of  $r(t)$  given  $r_0$  under the risk neutral probability measure. (3)
- c) Find a 94.8% confidence interval under the risk-neutral measure for the values of  $r(t)$ , when  $t = 0, t = 4, t = 8, t = 20$  and  $t = \infty$  given that  $r(0) = 6\%$ . Assume the following parameter values:  
 $\theta = 0.96\%$  ,  $\sigma = 2\%$  ,  $\alpha = 12\%$  .  
 Comment on your answer.  $\Phi(-1.94313) = 2.6\%$  (6)

[13]

**Q 8)** Your bank is asset sensitive and management wants to protect against loss from interest rate changes. Assets and liabilities are floating rate in nature. A bank is classified as asset sensitive if its rate sensitive assets are more than rate sensitive liabilities.

- a) Would an interest rate cap or floor serve as a better hedge? Explain. (2)
- b) Would a collar or reverse collar serve as a better hedge? Explain. (2)
- c) Why would the bank choose a collar or reverse collar over a cap or floor respectively? Explain. (2)

[6]

**Q 9)** A butterfly spread is the purchase of one call (put) at exercise price  $K_1$ , the sale of two calls (puts) at exercise price  $K_2$ , and the purchase of one call (put) at exercise price  $K_3$ .  $K_1$  is less than  $K_2$ , and  $K_2$  is less than  $K_3$  by equal amounts, and all calls (puts) have the same expiration date.

- a) Construct Separate tables showing how payoff varies with the stock price for the butterfly spreads using calls and puts separately. Also graph the payoff diagram to this strategy. At what stock price on expiration date would the butterfly spreads lead to maximum profit? (6)

- b) Assume that  $C_1$ ,  $C_2$  and  $C_3$ , are European call premiums with exercise prices of  $K_1$ ,  $K_2$  and  $K_3$ , respectively;  $P_1$ ,  $P_2$  and  $P_3$ , are European put premiums with exercise prices of  $K_1$ ,  $K_2$  and  $K_3$  respectively. All calls and puts have the same expiration date. Without using the put-call parity relationship, prove that  $2C_2 - C_1 - C_3 = 2P_2 - P_1 - P_3$  using no arbitrage argument. (2)
- c) Given below are the quoted prices observed for contracts on NSE Nifty at NSE as on 19<sup>th</sup> June 2009 at 2.50 PM.

Contract	Strike Price	Bid Price (Rs.)	Ask Price (Rs.)	Expiration Date
Call European	4300	89.00	90.00	June 25 2009
Call European	4350	63.60	68.00	June 25 2009
Call European	4400	44.40	45.00	June 25 2009
Put European	4300	65.00	68.00	June 25 2009
Put European	4350	82.00	90.50	June 25 2009
Put European	4400	119.00	121.30	June 25 2009

Bid price is the price at which you can sell the contract and the ask price is the price at which you can purchase the contract.

Using the relationship in (b), determine whether any arbitrage opportunities were existing for an arbitrageur on June 19 2009 at 2.50 PM? (2)

[10]

- Q 10)** A stock price follows geometric Brownian motion with an expected return of 18% and a volatility of 38%. The current price stock price is Rs. 60. One of the most important option strategies is straddle. Straddle involves buying a call and put with the same strike price and expiration date.
- a) When is straddle strategy effective? (1)
- b) A call with a strike price of Rs. 60 and expiration date in 3 months costs Rs. 6. A put with the same strike price and expiration date costs Rs. 4. What is the probability that the straddle lead to a loss? (3)
- [4]
- Q 11)** You are a derivative expert working for Master Derivative Co. The newly recruited CEO who has joined from the banking sector is struggling with few questions and has sought your help for the same. He would be glad if you can help answer a few of these.
- a) If the stock price falls and the call price rises, then what has happened to the call option's implied volatility? (1)
- b) If the time to maturity falls and the put price rises, then what has happened to the put option's implied volatility? (1)
- c) All else equal, will a call option with a high exercise price have a higher or lower delta than one with a low exercise price? (2)
- d) If the wheat harvest today is poor, would you expect this fact to have any effect on today's futures price for wheat to be delivered (postharvest) 2 years from now? Under what circumstances will there be no effect? (4)

- e) Evaluate the criticism that futures markets siphon off capital from more productive uses. (3)

[11]

- Q 12) Discuss three methods along with advantages/disadvantages from the perspective of an investment bank which may be used to mitigate the risk of default in a derivative contract.

[6]

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