

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

19th May 2010

Subject CT8 – Financial Economics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1) *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
- 2) *Mark allocations are shown in brackets.*
- 3) *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
- 4) *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q. 1)** The fund manager of Alpha Asset Management Company Limited has a well diversified portfolio that mirrors the performance of the BSE-SENSEX and is worth 8000 crores. The current value of the SENSEX is 16000 points. The fund manager would like to hedge the risk against a reduction of more than 10% in the value of the portfolio over the next six months. The risk free rate of interest is 6% per annum (with continuous compounding) and the volatility of the SENSEX is 25% per annum (with continuous compounding).
- What would the hedging cost be if the fund manager decides to hedge the risk using traded European put options on BSE-SENSEX? (6)
 - What are the alternate strategies available to the fund manager using traded European call options on BSE-SENSEX? Show that this strategy leads to the same result as strategy used in part (a) (4)
 - If the fund manager decides to hedge the risk by keeping part of the portfolio in the risk free securities, what should the initial position be? (3)
[13]
- Q. 2)**
- Explain the difference between hedging, speculation and arbitrage. (4)
 - Show that, if c is the price of an European call option on a future contract when the strike price is K and the maturity is T , and p is the price of an European put option on the same futures contract with the same strike price (K) and expiration date (T), then

$$c + Ke^{-rT} = p + F_0e^{-rT}$$
 Where F_0 is the current futures price and r is the risk-free rate per annum with continuous compounding. (4)
 - Suppose that a one-year futures price on Infosys stock is currently Rs.100. A one year call option and a one year put option on the futures (one-year futures on Infosys) with a strike price of Rs.95 are both currently priced at Rs.8 in the market. The risk free interest rate is 6% per annum (with continuous compounding). Identify an arbitrage opportunity available to an arbitrageur. (3)
[11]
- Q. 3)** A stock price is currently at Rs.100. Over the next two three-month periods it is expected to go up by 4% or down by 3%. The risk free rate of interest is 6% per annum with continuous compounding.
- What is the value of a six-month European call option with a strike price of Rs.102? (3)
 - What is the value of a six-month European put option with a strike price of Rs.102? (2)
 - Verify that the European call and European put prices satisfy the put-call parity. (2)

(d) If the put option in part (b) were American, would it be ever be optimal to exercise it early at any of the nodes on the tree? (3)
[10]

Q. 4) (a) What is a one-factor terms structure model? (1)

(b) What are the limitations of the one-factor terms structure models? (5)

(c) The cash prices of six-month and one-year zero coupon bonds are Rs. 94 and Rs. 89 respectively. A 1.5 year bond that will pay coupons of Rs. 4 every six months currently sells for Rs. 94.84. A two-year bond that will pay coupons of Rs. 5 every six months currently sells for 97.12. Calculate the six-month, one-year, 1.5-year and two-year spot (zero) rates. All bonds have face value of Rs. 100. (6)
[12]

Q. 5) (a) What are different ways in which a credit default impacts the contracted payment stream? (2)

(b) Define the term credit event. (2)
[4]

Q. 6) Ram's utility $U(w)$ for a given level of wealth 'w' is determined by the following function:

$$U(w) = \begin{cases} 4\left(\frac{w}{10000}\right)^2, & w \leq 5,000 \\ \frac{3w}{5000} - 2, & 5,000 < w \leq 10,000 \\ \log_{10} w, & w > 10,000 \end{cases}$$

(i) State the principle of non-satiation in the context of an investor. (1)

(ii) Is Ram a non-satiated investor? (2)

(iii) Define the following types of investors including the respective behavior of their utility function (3)

- risk averse investor
- risk seeking investor
- risk neutral investor

(iv) What can you say about Ram's attitude towards risk at different levels of wealth? (4)
[10]

Q. 7) You are constructing a two-factor model for return on stocks. For this purpose you have chosen two indices BSE Sensex (F_1) and ET Banks Index (F_2). F_1 is widely regarded as the market index.

(i) Your close friend, an actuary, makes the following statement.

“For applications of multi-index models to portfolio selection problems it is convenient if the factors used are uncorrelated (or orthogonal).”

F_1, F_2 in their current form are not orthogonal. Describe how would you transform the set (F_1, F_2) into an orthogonal set. (3)

(ii) Your friend is a big fan of simple models. He encourages you to simplify the return R_i on security ‘ i ’ to be modeled as follows

$$R_i = a_i + b_i F_1 + e_i$$

where:

a_i, b_i are constants and e_i is the random residual unexplained by market movement.

Derive expressions for the following

- V_i the variance of the return on security i
- C_{ij} the covariance of returns on the securities i and j
- V_p the variance of portfolio returns assuming there are ‘ n ’ securities in the market and equal amount $\frac{1}{n}$ is invested in each

Define all the terms and state the assumptions made. (6)

(iii) Further, assume that

$$DR = \frac{1}{n} \sum_{i=1}^n \text{Var}(e_i) \quad \text{and}$$

$$\beta_p = \frac{1}{n} \sum_{i=1}^n b_i$$

Prove using result in (ii) that

$$V_p = \frac{1}{n} DR + \beta_p^2 * \text{Variance of BSE Sensex} \quad (2)$$

(iv) What happens to the variance of the portfolio in the limiting case where $n \rightarrow \infty$? Interpret the result. (3)

[14]

- Q. 8)** (i) What is the market price of risk under CAPM? Define all the terms you use. (2)

With discovery of water on moon, three countries have set up colonies on it. Investment market on moon is in its nascent stage and investors have only three risky assets to choose from. The annual returns from these three assets are as in the table below depending on which country manages to send more settlers:

	<i>More Indians</i>	<i>More Americans</i>	<i>More British</i>	<i>Market Cap (bn)</i>
<i>Food Chain</i>	40%	20%	10%	100
<i>Bank</i>	10%	15%	20%	50
<i>Arms Dealer</i>	0%	60%	20%	25

India has a distinct cost advantage. The respective probabilities of three different scenarios is as follows

	<i>Probability</i>
<i>More Indians</i>	0.80
<i>More Americans</i>	0.15
<i>More British</i>	0.05

Annual rate of return on Moon’s government bond stands at 20% pa for all terms.

- (ii) Calculate the market price of risk for Moon’s investment market (5)
[7]

- Q. 9)** You overhear the following statement while eating lunch with your overly curious colleague

“Numerous studies have shown that the stock market over-reacts to certain events and under-reacts to other events.”

Your colleague considers you expert in stock market and expects you to be able to explain the statement using examples.

Cite 2 examples each of under-reaction and over-reaction of stock markets to satisfy your friends curiosity.

[4]

- Q. 10)** A(t) follows the stochastic equation

$$A(t) = S(t)e^{(R-r)(T-t)}$$

and you are also given that

$$\frac{dS(t)}{S(t)} = a dt + b dZ(t)$$

where $Z(t)$ is standard Brownian motion under the real world measure P .

- (i) Apply Ito's formula to derive an SDE satisfied by $A(t)$ (5)
 - (ii) Define a martingale in words (1)
 - (iii) Under what conditions will $A(t)$ be a martingale. (2)
- [8]**

Q. 11) Stocks offer an expected rate of return of 18% with a standard deviation of 22%. Gold offers an expected return of 10% with a standard deviation of 30%. Gold offers an return of 10% with a standard deviation of 30%.

- a. In light of inferiority of gold with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so. (3)
- b. Given the data above, reanswer (a) with the additional assumption that the correlation coefficient between gold and stocks equals one. Draw a graph illustrating why one would or would not hold gold in one's portfolio. Could this set assumptions for expected returns, standard deviations, and correlation represent an equilibrium for the security market? (4)

[7]

