INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

19th November 2010

Subject ST6 — Finance and Investment B

Time allowed: Three hours (9.45* – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***

2. **** You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.***

3. ***You must not start writing your answers in the answer sheet until instructed to do so by the supervisor***

4. ***The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***

5. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

6. ***Mark allocations are shown in brackets.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) Consider the following options portfolio. On November 1 2010 at 11 AM, you write a November expiration call option on NSE Nifty futures with exercise price of 4300. On the same date and time, you also write a November Nifty futures put option with exercise price of 4300. Assume that the call Nifty futures option, put Nifty futures option and Nifty futures have the same expiration date (November 26 2010). The following are the quoted prices of the above mentioned options and futures on November 1 2010 at 11 AM. All the options are of European style.

	Exercise price	Expiration Date	Bid Price	Ask Price
November Nifty Futures Call Option	4300	November 26, 2010	89	90
November Nifty Futures Put Option	4300	November 26, 2010	65	68
November Nifty Futures	-	November 26, 2010	4325	4326

Assume that only the above mentioned options and futures are being traded in the market. The bid price is the price at which you can sell and ask price is the price at which you can buy.

a. Graph the payoff of this portfolio at option expiration as a function of spot value of NSE Nifty at that time. (3)

b. What will be the profit/loss on this position if the spot value of NSE Nifty is 4100 on the option expiration date? What if the spot value of NSE Nifty is 4500? (2)

c. At what two values of NSE Nifty on expiration date will you just break-even on your investment? (2)

d. What kind of bet you are making as an investor; that is, what must you believe about spot value of NSE Nifty to justify your position? (2)

e. What must be the risk-free rate of interest on November 1 2010 such that you are not able to exploit any arbitrage opportunity? Assume that risk-free borrowing rate is the same as risk-free lending rate. (3)

f. If the risk-free rate of interest (both borrowing and lending) is 5% per annum with continuous compounding, what must the quoted bid and ask values of NSE Nifty on November 1 2010 such that there does not exist any arbitrage opportunity? Assume that NSE Nifty index is traded in the spot market and short selling is allowed. (3)

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Q. 2) Consider two points in time U and T, such that $U < T$ and the payoff (Π) of a caplet at time T is given as:

$$\Pi_T = (T - U) Max\{M(U,T) - K, 0\}$$

where $M(U,T)$ denotes the MIBOR rate contracted at time U for the period [U,T] and the cap rate K. $M(U,T)$ is expressed with a compounding period equal to U-T.

Show that the price of the caplet at time t ($0 \le t \le U$) can be replaced using the price of a portfolio (at time t) of European put options written on zero-coupon bonds. **[8]**

Q. 3) According to Heath-Jarrow-Morton (HJM) model, the zero-coupon bond price has the following stochastic differential equation under Q:

$$dP(t,T) = r(t)P(t,T)dt + v(t,T)P(t,T)dz(t)$$

where z denotes a Q-wiener process.

Consider the above HJM model and if T_2 is very close to T_1 (that is, T_2 ® T_1), compute $E^Q\left[\frac{1}{P(T_1,T_2)} \mid F_t\right]$ where $t \le T_1 \le T_2$

For the pricing of which financial instrument the expression $E^Q\left[\frac{1}{P(T_1,T_2)} \mid F_t\right]$ is connected? **[8]**

Q. 4) Alpha Limited manages a Rs. 302.50 million equity portfolio benchmarked to the NSE Nifty index portfolio. Over the past one year, the NSE Nifty index has appreciated 70%. The fund manager of Alpha Limited believes that the market is overvalued when measured by traditional economic indicators. He is concerned about maintaining the excellent gains the portfolio has experienced in the past one year but recognizes that the NSE Nifty index can still move above its current 6050 level.

The fund manager is considering the following option strategy:

- Protection for the portfolio can be attained by purchasing a NSE Nifty European put option with a strike price of 6000.
- The put can be financed by selling two European call options with an exercise price of 6400.

The information in the following table describes the two options:

Characteristics	6400 Call	6000 Put
Option price	52.48	103.29
Option Implied Volatility	23.5%	20%
Option's Delta	0.229	-0.403
Time to Option Expiration	30 Days	30 Days
NSE Nifty Historical 30-Day Volatility	22%	22%

a. How many contracts of 6000 puts and 6400 calls should be purchased such that in the event of the decline in the spot value of NSE Nifty from the current level of 6050, the minimum payoff of the combined portfolio (the equity portfolio plus the options) is Rs. 300 million on the expiration date of the options? The contract multiplier on NSE Nifty options is Rs. 50. (2)

b. What is the delta of the combined portfolio [the equity portfolio plus the position in the options established in part (a)]? What is the effect on the value of the combined portfolio, if on the next day of the establishment of the above portfolio, the value of NSE Nifty index (i) rises, (ii) falls? (2)

c. Describe the potential returns of the combined portfolio [the equity portfolio plus the position in the options established in part (a)] if on expiration date the NSE Nifty index has:

(i) risen by 8%

(ii) remained at 6050

(iii) declined by 8%. (4.5)

d. Discuss the effect on the delta of each option on the expiration date as the NSE nifty index approaches the level for each of the potential outcomes listed in part (c). (4.5)

e. Evaluate the pricing of each option (6400 call and 6000 put) in relation to the volatility data provided. (1)

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Q. 5) a) Why is the beta of a call option on Infosys stock greater than the beta of a share on Infosys? (4)

b) Suppose the current spot value of NSE Nifty is 6000. Is the following statement true or false? Explain your answer.

"The beta of a call option on the NSE Nifty index with an exercise price of 6000 is greater than the beta of a call option on the NSE Nifty index with an exercise price of 6100." (4)

c) All else being equal, is a put option on a high-beta stock worth more than the one on a low beta stock? Explain your answer. The stocks have identical firm-specific risk. (1)

d) All else being equal, is a call option on a stock with a lot of firm-specific risk worth more than the one on a stock with little firm-specific risk? Explain your answer? The betas of the two stocks are equal. (1)

e) Delta grows Number 1 wheat and would like to hedge the value of the coming harvest. However, the futures contract is traded on the Number 2 grade of wheat. Suppose that Number 2 wheat sell for 95% of the price of number 1 wheat. If Delta grows 50,000 Kg, and each futures contract calls for delivery of 2000 Kg of wheat, how many contracts should Delta go long or short in to hedge his position? (2)

f) The India yield curve is flat at 9% (with annual compounding) and the US yield curve is flat at 4% (with annual compounding).The current exchange rate is Rs. 50 per dollar. What will be the swap rate on an agreement to exchange currency over a four-year period? The swap will call for the exchange of 500,000 dollars for a given number of rupees in each year. (4)

g) To hedge interest rate risk, what type of interest rate swap would be appropriate for a corporation holding fixed rate long-term financial assets that is funded with floating rate bonds? (1)

[17]

Q. 6) You run a regression of the yield of Beta company's 20-year bond on the 20-year Indian Treasury benchmark's yield using month-end data for the past years. You have found the following regression result:

$$Y_b = 0.40\% + 1.10Y_T$$

where Y_b is the yield per annum (with semi-annual compounding) on the bond of Beta Company and Y_T is the yield per annum (with semi-annual compounding) on the Indian Treasury bond.

The duration and the modified duration (with annual compounding) of the 20-year Indian Treasury bond are 12.7308 years and 12 years respectively. The duration of the bond of Beta company is 11.5 years.

a. What is the yield per annum (with semi-annual compounding) on the 20-year Indian Treasury bond? (1)

b. What is the yield per annum (with semi-annual compounding) on the 20-year bond of Beta company? (1)

c. What is the modified duration (with annual compounding) of the 20-year bond of Beta company? (1)

d. Calculate the percentage change in the price of the 20-year Indian Treasury bond, assuming a 75-basis-point increase in the yield per annum (with semi-annual compounding) in the 20-year Indian Treasury bond. (2)

e. Calculate the percentage change in the price of the 20-year bond of Beta company, assuming a 75-basis-point increase in the yield per annum (with semi-annual compounding) in the 20-year Indian Treasury bond. (3)

f. You are managing a portfolio consisting of Rs. 400 million investment in the 20-year Indian Treasury bond and Rs. 100 million investment in the 20-year bond of Beta company. Suppose that you decide to hedge the portfolio against the fluctuation in interest rates over the next three months by using Treasury bond futures. The Treasury bond futures price (expiring in six months) is currently 105 and the cheapest to deliver bond will have a modified duration (with annual compounding) of 9 years in three months. Each futures contract is for the delivery of Rs. 100,000 face value of bonds. How should you immunize the portfolio against changes in interest rate over the next three months? Assume that both the bonds (20-year Treasury bond and 20-year Beta bond) are not expected to make any payments over the next three months. (5)

[13]

Q. 7) A company has issued 2-year, 4-year and 6-year bonds with a coupon rate of 8% payable annually. The yields on the bonds (with continuous compounding) are 8.5% per annum, 8.75% per annum and 9% per annum respectively. The risk-free yield curve is flat at 6% per annum with continuous compounding. Assume that defaults can take place halfway through each year and that the recovery rate is 50%. The company's risk-neutral default rates per year are Q_1 for years 1 and 2, Q_2 for years 3 and 4 and Q_3 for years 4 and 5.

Estimate Q_1, Q_2 and Q_3.

[9]

Q. 8) How does a 6-year *n*th-to-default credit default swap work?

Consider a basket of 200 reference entities where each reference entity has a probability of defaulting in each year of 0.5%. As the default correlation between the reference entities increases, what would you expect to happen to the value of the swap when (a) $n = 1$ and (b) $n = 50$. Explain your answer.

[5]

Q. 9) Consider a two-dimensional Black-Scholes market. The market consisting of a risk-free asset, B, with P-dynamics given by:

$$\begin{cases} dB_t = rB_t dt \\ B_0 = 1 \end{cases}$$

and two stocks S^1 and S^2 with P-dynamics given by:

$$\begin{cases} dS_t^1 = \mu S_t^1 dt + \rho S_t^1 dV_t + \sigma S_t^1 dW_t \\ dS_t^2 = \alpha S_t^2 dt + \beta S_t^2 dV_t \end{cases}$$

where W and V are two independent P-Wiener processes and m, r, s, a and b are assumed to be constant. Using Girsanov's theorem, show that the stochastic differential equations of S_t^1 and S_t^2 under Q-Wiener process (risk-neutral measure) are given as:

$$\begin{cases} dS_t^1 = rS_t^1 dt + \rho S_t^1 d\tilde{V}_t + \sigma S_t^1 d\tilde{W}_t \\ dS_t^2 = rS_t^2 dt + \beta S_t^2 d\tilde{V}_t \end{cases}$$

where $\tilde{V}$ and $\tilde{W}$ are two independent Q-Wiener process.

[8]

Q. 10) Consider the standard Black-Scholes model. Consider the portfolio corresponding to:

- buying a European call with exercise price K_1 and selling a European call with exercise price K_2
- buying a European put with exercise price K_2 and selling a European put with exercise price K_1

where $K_1 < K_2$.

All options are written on the same underlying asset and have the same date T. Show that the arbitrage free price of the above portfolio at time t is given by $e^{-r(T-t)}(K_2 - K_1)$.

[3]

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