

**BACHELOR OF SCIENCE (B.Sc.)****Term-End Examination****December, 2005****PHYSICS****PHE-14 : MATHEMATICAL METHODS IN  
PHYSICS-III**

Time : 2 hours

Maximum Marks : 50

**Note :** Attempt **all** questions. The marks for each question are indicated against it. Symbols have their usual meanings.

**1.** Attempt any **five** parts :

2×5=10

(a) Write the equations

$$(a_{11} - \lambda) x_1 + a_{12} x_2 + a_{13} x_3 = 0$$

$$a_{21} x_1 + (a_{22} - \lambda) x_2 + a_{23} x_3 = 0$$

$$a_{31} x_1 + a_{32} x_2 + (a_{33} - \lambda) x_3 = 0$$

as an eigenvalue equation.

- (b) Write down the quadratic form corresponding to the following symmetric matrix :

$$\begin{pmatrix} 3 & 3 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

- (c) Show that the dot product of two 3-D vectors is a scalar.
- (d) Show by operating on a rectangle that  $m_x C_4 = \sigma_v$ .
- (e) Obtain the analytic function whose real part is  $u(x, y) = x$
- (f) Determine the type of singularity and the region of convergence of  $\frac{e^z}{(z-1)^2}$ .
- (g) Show that  $f(x) = 1, x > 0$  cannot be represented by a Fourier integral.
- (h) Plot  $J_0(x)$  as a function of  $x$ .

2. Determine the eigenvalues and eigenvector of the hermitian matrix

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

10

OR

- (a) Show that electrical conductivity tensor  $\sigma$  transforms as a contravariant tensor of rank 2. 4
- (b) Show that the set of all matrices of order  $m \times n$  is a group under addition of matrices. Is this group abelian? 6

3. Attempt any **two** parts : 2x5=10

- (a) If  $C$  is a circle  $|z| = \pi$  described in the positive sense and

$$g(z_0) = \oint_C \frac{z^3 - 6}{z - z_0} dz$$

calculate  $g(3)$ .

- (b) Evaluate the integral  $\oint_C \frac{dz}{4 + z^2}$  where  $C$  is a circle  $|z| = 3$ .
- (c) Determine the Laurent series for

$$f(z) = \frac{1}{z^2 - 4z + 3} \text{ valid for } 1 < |z| < 3.$$

4. (a) Calculate the Laplace transform of the function

$$f(t) = \begin{cases} 1, & 0 < t < \pi/2 \\ 0, & \text{otherwise} \end{cases} \quad 5$$

- (b) Using Laplace transforms, solve the initial value problem

$$y'' + 5y' + 4y = 0, \quad y(0) = y'(0) = 0. \quad 5$$

**OR**

Consider an infinite metal plate placed in the  $xy$  plane. Its edge along  $y$ -axis is maintained at temperature  $0^\circ \text{C}$  and the temperature in the edge along  $x$ -axis is given by

$$T(x, 0) = \begin{cases} 50^\circ \text{C}, & 0 < x < 2 \\ 0^\circ \text{C}, & x > 2 \end{cases}$$

Determine the steady-state temperature distribution of the plate using Fourier transform method. 10

5. Attempt any **one** part : 10

(a) The generating function for Hermite polynomial is

$$g(x, t) = e^{2xt-t^2} = \sum_{n=0}^{\infty} H_n(x) \frac{t^n}{n!}$$

Show that  $H'_n(x) = 2nH_{n-1}(x)$  and

$$H_n(x) = 2xH_{n-1}(x) - H'_{n-1}(x).$$

(b) Expand the function

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & -1 < x < 0 \end{cases}$$

in terms of Legendre polynomials.