Reg. No. : | | | | | | | | | | | |

T 3319

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2008.

Third Semester

(Regulation 2004)

Civil Engineering

MA 1201 — MATHEMATICS — III

(Common to all branches of B.E./B.Tech. Except Bio-Medical Engineering)

(Common to B.E. (Part-Time) Second Semester Regulation 2005)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Form a partial differential equation by eliminating arbitrary constants a and b from $z = (x+a)^2 + (y+b)^2$.

2. Solve : $\left(D^2 - 2DD' + D'^2\right) z = 0$.

3. Determine the value of a_n in the Fourier series expansion of $f(x) = x^3$ in $-\pi < x < \pi$.

4. Find the root mean square value of $f(x) = x^2$ in the interval $(0, \pi)$.

5. Classify the partial differential equation $3u_{xx} + 4u_{xy} + 3u_y - 2u_x = 0$.

6. The ends A and B of a rod of length 10 cm long have their temperature kept at 20°C and 70°C. Find the steady state temperature distribution on the rod.

7. State the Fourier integral theorem.

8. State the convolution theorem of the Fourier transform.

9. Find $Z\{n\}$.

10. Form the difference equation from $y_n = a + b3^n$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Form a partial differential equation by eliminating arbitrary functions from $z = x f(2x+y) + g(2x+y)$.

(ii) Solve : $p^2 y(1+x^2) = qx^2$.

(iii) Find the singular integral of $z = px + qy + p^2 - q^2$. (6 + 5 + 5)

Or

(b) (i) Solve : $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$. (8)

(ii) Solve : $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = e^{x+2y} + 4\sin(x+y)$. (8)

12. (a) (i) Expand in Fourier series of $f(x) = x \sin x$ for $0 < x < 2\pi$ and deduce the result $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{3.7} - \ldots\ldots\infty = \frac{\pi - 2}{4}$. (10)

(ii) Obtain half range sine series for $f(x) = x$ in $0 < x < l$ and deduce the series $\sum_1^\infty \frac{1}{n^2} = \pi^2/6$. (6)

Or

(b) (i) Find the Fourier series of periodicity 3 for $f(x) = 2x - x^2$ in $0 < x < 3$. (8)

(ii) The table of values of the function $y = f(x)$ is given below :

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y :	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Find a Fourier series upto the second harmonic to represent $f(x)$ interms of x. (8)

13. (a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point of the string at a distance x from one end at any time t. (16)

Or

(b) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite length without introducing appreciable error. If the temperature at short edge $y = 0$ is given by

$$u = \begin{cases} 20x & \text{for } 0 \le x \le 5 \\ 20\ (10 - x) & \text{for } 5 \le x \le 10 \end{cases}$$

and all the other three edges are kept at 0°C. Find the steady-state temperature at any point of the plate. (16)

14. (a) Find Fourier transform of $f(x) = 1 - |x|$ if $|x| < 1$

$= 0$ if $|x| > 1$

and hence find the value of $\int_0^\infty \left(\frac{\sin t}{t}\right) dt$ and $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$. (16)

Or

(b) Find Fourier sine transform and Fourier cosine transform of e^{-ax}, $a > 0$.

Hence evaluate $\int_0^\infty \frac{x^2}{(a^2 + x^2)^2} dx$ and $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$. (16)

15. (a) (i) Find the z-transform of $\left\{\frac{1}{n}\right\}$ and $\left\{\cos\frac{n\pi}{2}\right\}$. (4 + 4)

(ii) Find the inverse z-transform of $\frac{8z^2}{(2z-1)(4z-1)}$ by using convolution theorem. (8)

Or

(b) (i) Find the z-transform of $\{a^n\}$ and $\{na^n\}$. (4 + 4)

(ii) Solve : $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given $y_0 = y_1 = 0$, using z-transform. (8)