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B.E/B.Tech. DEGREE EXAMINATION, APRIL/MAY 2009.

FIRST SEMESTER

MA 12 — MATHEMATICS — I

(Common to all B.E/B.Tech)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Eigen values of the matrix A^{-1} if $A = \begin{pmatrix} 1 & 5 \\ 0 & 4 \end{pmatrix}$.

2. Show that the matrix $P = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

3. Find the equation of the sphere which has its centre at $(-1, 2, 3)$ and touches the plane $2x - y + 2z = 6$.

4. Find the equation to the cone with vertex at the origin and passes through the curve $ax^2 + by^2 + cz^2 = 1, lx + my + nz = p$.

5. Find the envelope of $x\cos\alpha + y\sin\alpha = p$ where α is the parameter.

6. Find the radius of curvature at any point of $y = \cosh\left(\frac{x}{c}\right)$.

7. If $u = \log\left(x^2 + xy + y^2\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$.

8. Find $\frac{du}{dt}$ if $u = x^2 + y^2, x = at^2, y = 2at$.

9. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\cos\theta} r^2 dr d\theta$.

10. Change the order of integration of $\int_0^a \int_y^a f(x,y)\, dxdy$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(ii) Using Cayley–Hamilton theorem find A^{-1} if $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$.

Or

(b) (i) Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ to a canonical form through an orthogonal transformation.

(ii) If the matrices A and B are orthogonal prove that AB is orthogonal.

12. (a) (i) Find the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x - 2y - 6z + 5 = 0$ which are parallel to the plane $x + 4y + 8z = 0$.

(ii) Find the equation to the right circular cylinder of radius 2 units and whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$.

Or

(b) (i) Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$.

(ii) Find the equation of the right circular cone generated by the straight line drawn from the origin to cut the circle through the three points $(1, 2, 2)$, $(2, 1, -2)$ and $(2, -2, 1)$.

13. (a) (i) Show that the radius of curvature ρ at any point (x, y) of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ satisfies $\rho^3 = 27axy$.

(ii) Find the evolute of the parabola $y^2 = 4ax$.

Or

(b) (i) Find the centre of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

(ii) Considering the evolute of a curve as the envelope of its normals, find the evolute of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

14. (a) (i) If $u = \frac{y^2}{2x}$ and $v = \frac{x^2 + y^2}{2x}$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

(ii) Prove that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube.

Or

(b) (i) Find the points on the surface $z^2 = xy + 1$ whose distance from the origin is minimum.

(ii) Expand e^{xy} at $(1, 1)$ in powers of x and y as far as the term of second degree.

15. (a) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1 + x^2 + y^2}$.

(ii) Evaluate $\int \int r^3 dr d\theta$ over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.

Or

(b) (i) Change the order of integration of $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dxdy$ and hence evaluate the same.

(ii) Show that $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dzdydx = \frac{8}{3}\log^2 - \frac{19}{9}$.