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Printed Pages : 4 **MCA – 124**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1452 Roll No.

MCA

(SEM. II) EXAMINATION, 2006-07

COMBINATORICS & GRAPH THEORY

Time : 3 Hours] *[Total Marks : 100*

Note : *Attempt **all** the questions.*

1. Answer any **four** parts of the following:- **5×4=20**

(a) Determine the number of positive integers n where: $1 < n \leq 100$ and n is not divisible by 2, 3, or 5?

(b) Find the sequence corresponding to the generating function $f(x) = (1 + x)^{-n}$, where n is positive integer.

(c) Solve the recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = (n+1)2^n$.

(d) Show that no simple graph can have the degrees of all its vertices distinct.

(e) Define Euterian and Hamilton graph. Draw a graph with six vertices which is

(i) Hamiltonian and non-eulerian

(ii) Eulerian and non-hamiltonian

(f) Discuss the traveling salesman problem.

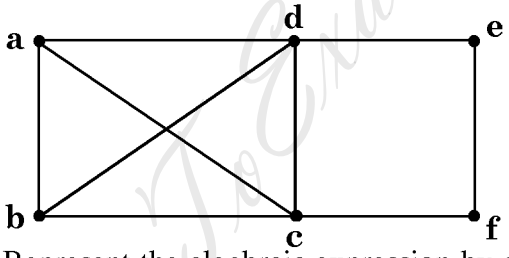
2. Answer any **four** parts of the following **5×4=20**

(a) Define central tree, rooted and binary tree. Prove that a simple graph G is a tree if and only if there is one path between every pair of vertices.

(b) Prove that a tree having n vertices has exactly (n-1) edges.

(c) Let G be a graph having V vertices and E edges and K components, where each component is a tree. Obtain a formula in terms of V, E and K.

(d) Define spanning tree and minimal spanning tree.Draw three spanning trees of the graph.



(e) Represent the algebraic expression by a binary rooted tree.

$(7+a)\%\ 8\ x\ (a-b)^{3}$.

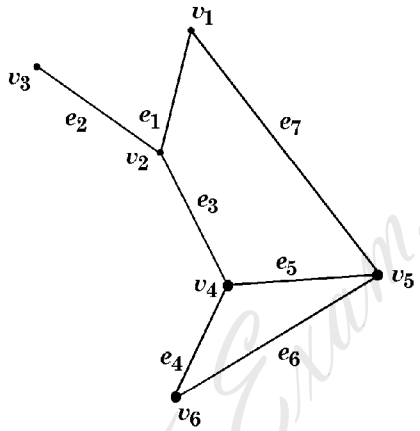
(f) Write short note on connectivity and seperativity. Network flow Nin-cut theory.

3. Answer any **two** of the following: **10×2=20**

(a) Prove that for any connected planar graph G, V-e+r=2. Where v, e and r are the number of vertices, edges and regions of the graph respectively.

(b) Define the dual of a graph G. Prove that a graph G has a dual G if and only if it is planar. Write properties of Graph and dual graph.

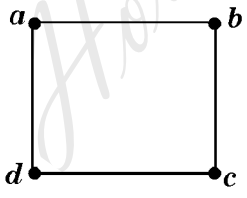
(c) Define incidence and adjancy matrices of a graph. Find incidence and adjancy matrics of the following graph.



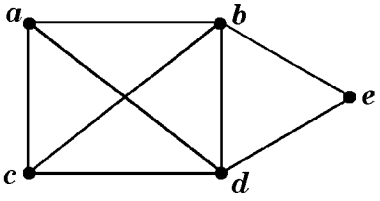
4. Answer any **two** of the following :- **10×2=20**

(a) Define chromatic polynomial. Find the chromatic polynomial and chromatic number of the following graphs.

(i)

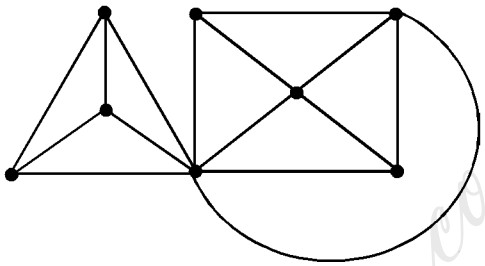


(ii)



(b) Explain the four colour problem. Show that vertices of a planar graph with less than 30 edges is 4-colourable.

(c) Explain connectivity and seperability of a graph. What is the maximum vertex connectivity and edge connectivity for the graph shown in the following figure.



5. Answer any **two** of the following:- **10×2=20**

(a) Define abborescence graph. Write down the procedure to obtain the expression in polish notation.

(b) State and prove Cayley's theorem.

(c) In how many distinct ways can we 4 colour vertices of a regular hexagon which is free to move in space.