

**GUJARAT TECHNOLOGICAL UNIVERSITY**

B.E. Sem-IV Examination June- 2010

**Subject code: 140001****Subject Name: Mathematics-4****Date: 15 / 06 / 2010****Time: 10.30 am – 01.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Do as directed.

(14)

- (a) Find the value of  $\text{Re}(f(z))$  and  $\text{Im}(f(z))$  at the indicated point where

$$f(z) = \frac{1}{1-z} \text{ at } 7 + 2i.$$

- (b) Find the value of the derivative of  $\frac{z-i}{z+i}$  at  $i$ .

- (c) Find an upper bound for the absolute value of the integral  $\int_C e^z dz$ , where  $C$  is the line segment joining the points  $(0,0)$  and  $(1, 2\sqrt{2})$ .

- (d) Evaluate  $\oint_C \frac{dz}{z^2+1}$ , where  $C$  is  $|z+i|=1$ , counterclockwise.

- (e) Develop  $f(z) = \sin^2 z$  in a Maclaurin series and find the radius of convergence.

- (f) Define : (i) Singular point (ii) Essential singularity  
(iii) Removable singularity (iv) Residue of a function

- (g) If  $f(x) = \frac{1}{x}$ , find the divided differences  $[a,b]$  and  $[a,b,c]$ .

Q.2 (a) Evaluate  $\int_0^1 e^{-x^2} dx$  by the Gauss integration formula with  $n=3$ . (03)

- (b) Compute  $f(9.2)$  from the following values using Newton's divided difference formula. (04)

|      |          |          |          |          |
|------|----------|----------|----------|----------|
| x    | 8        | 9        | 9.5      | 11.0     |
| f(x) | 2.079442 | 2.197225 | 2.251292 | 2.397895 |

- (c) (i) Find the positive root of  $x = \cos x$  correct to three decimal places by bisection method. (03)

- (ii) Solve the following system of equations using partial pivoting by Gauss-elimination method. (04)

$$8x_2 + 2x_3 = -7$$

$$3x_1 + 5x_2 + 2x_3 = 8$$

$$6x_1 + 2x_2 + 8x_3 = 26$$

**OR**

- (c) (i) Find the dominant eigen value of  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  by power method and hence (03)

find the other eigen value also. Verity your results by any other matrix theory.

- (ii) Solve the following system of equations by Gauss- seidal method. (04)

$$10x_1 + x_2 + x_3 = 6$$

$$x_1 + 10x_2 + x_3 = 6$$

$$x_1 + x_2 + 10x_3 = 6$$

- Q.3 (a) Determine the interpolating polynomial of degree three using Lagrange's interpolation for the table below : (04)

|      |    |   |   |    |
|------|----|---|---|----|
| x    | -1 | 0 | 1 | 3  |
| f(x) | 2  | 1 | 0 | -1 |

- (b) Evaluate  $\int_0^3 \frac{dx}{1+x}$  with n=6 by using Simpson's  $\frac{3}{8}$  rule and hence calculate (05)

log2. Estimate the bound of error involved in the process.

- (c) Using improved Euler's method, solve  $\frac{dy}{dx} + 2xy^2 = 0$  with the initial condition  $y(0)=1$  and compute  $y(1)$  taking  $h = 0.2$ . Compare the answer with exact solution. (05)

**OR**

- Q.3 (a) Find an iterative formula to find  $\sqrt{N}$  (where N is a positive number) and hence find  $\sqrt{5}$ . (04)

- (b) Compute cosh 0.56 from the following table and estimate the error. (05)

|        |          |          |          |          |
|--------|----------|----------|----------|----------|
| x      | 0.5      | 0.6      | 0.7      | 0.8      |
| cosh x | 1.127626 | 1.185465 | 1.255169 | 1.337435 |

- (c) Apply Runge-Kutta method of fourth order to calculate  $y(0.2)$  given  $\frac{dy}{dx} = x+y$ ,  $y(0) = 1$  taking  $h=0.1$  (05)

- Q.4 (a) Find and plot all roots of  $\sqrt[3]{8i}$ . (03)

- (b) Find out (and give reason) whether  $f(z)$  is continuous at  $z=0$  if (03)

$$f(z) = \frac{\operatorname{Re}(z^2)}{|z|}, \quad z \neq 0$$

$$= 0, \quad z = 0$$

- (c) Using residue theorem, evaluate  $\oint_c \frac{z^2 \sin z}{4z^2 - 1} dz$ ,  $c : |z| = 2$  (04)

- (d) (i) Expand  $f(z) = \frac{1-e^z}{z}$  in Laurent's series about  $z=0$  and identify the singularity. (02)

- (ii) Find all solutions of  $\sin z = 2$ . (02)

**OR**

- Q.4 (a) Solve the equation  $z^2 - (5+i)z + 8 + i = 0$ . (03)

- (b) Show that if  $f(z)$  is analytic in a domain D and  $|f(z)| = k = \text{const.}$  in D, then  $f(z) = \text{const.}$  in D. (03)

- (c) Find all Taylor and Laurent series of  $f(z) = \frac{-2z+3}{z^2-3z+2}$  with center 0. (04)

- (d) (i) Find the center and the radius of convergence of the power series (02)

$$\sum_{n=0}^{\infty} (n+2i)^n z^n$$

(ii) State and prove Cauchy's residue theorem. (02)

Q.5 (a) Find and sketch the image of region  $x \geq 1$  under the transformation  $w = \frac{1}{z}$  (03)

(b) Using the residue theorem, evaluate  $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$  (03)

(c) Evaluate  $\int_C \operatorname{Re}(z^2) dz$ , where C is the boundary of the square with vertices 0, i,  $1 + i$ , 1 in the clockwise direction. (04)

(d) (i) State and prove cauchy integral theorem. (02)

(ii) Determine a and b such that  $u = ax^3 + bxy$  is harmonic and find a conjugate harmonic. (02)

**OR**

Q.5 (a) Define Mobius transformation. Determine the mobius transformation that maps  $z_1 = 0, z_2 = 1, z_3 = \infty$  onto  $w_1 = -1, w_2 = -i, w_3 = 1$  respectively. (03)

(b) Using contour integration, show that  $\int_0^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$  (03)

(c) Evaluate  $\oint_C \frac{e^z}{z(1-z)^3} dz$ , where C is (a)  $|z| = \frac{1}{2}$  (b)  $|z-1| = \frac{1}{2}$ . (04)

(d) Check whether the following functions are analytic or not. (04)

(i)  $f(z) = z^{\frac{5}{2}}$  (ii)  $f(z) = \bar{z}$

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