ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2008
STATISTICS AND NUMERICAL TECHNIQUES
SEMESTER - 3

Time : 3 Hours]   [Full Marks : 70

Graph sheets are provided at the end of the booklet.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) Two unbiased coins are tossed. What is the probability of obtaining '3 heads' ?

   a) $\dfrac{1}{2}$ b) $\dfrac{1}{3}$

   c) $\dfrac{1}{4}$ d) 0.

   ii) If $\bar{x}$ be the arithmetic mean of the values x_i weighted by f_i ($i = 1, 2, \ldots, n$), then

   $$\sum_{i=1}^{n} f_i \left(x_i - \bar{x} \right) = 0.$$

   a) True   b) False.

   iii) What is the Standard Error of the sample mean for a random sample of size n drawn with replacement (σ = s.d., N = population size, n = sample size) ?

   a) $\dfrac{\sigma}{\sqrt{n}} \dfrac{\sqrt{N-n}}{\sqrt{N-1}}$ b) $\dfrac{\sigma}{\sqrt{n}}$

   c) $\sigma \dfrac{\sqrt{N-n}}{N-1}$ d) $\sigma \dfrac{N-n}{\sqrt{N-1}}$

   iv) What is the probability that a leap year selected at random contains 53 Sundays ?

   a) $\dfrac{1}{366}$ b) $\dfrac{2}{366}$

   c) $\dfrac{2}{7}$ d) $\dfrac{3}{7}$.

v)   For two events A, B, $P(B / A) =$

    a) $P(AB) / P(B)$ b) $P(AB) / P(A)$

    c) $P(B) / P(A)$ d) $P(A) * P(B)$.   [   ]

vi)   The number of significant figures in 0·0128742 is

    a)   five   b)   six

    c)   seven   d)   three.   [   ]

vii)   If $f(x)$ is a polynomial of degree n, the nth order difference is a constant.

    a)   True   b)   False.   [   ]

viii)   Relation between mean and variance of a random variable following binomial distribution is

    a)   Mean = Variance   b)   Mean > Variance

    c)   Mean < Variance   d)   Mean = Variance^2 .   [   ]

ix)   For a distribution, mean, median and mode are found to be equal. What kind of distribution is the most possibility ?

    a)   Binomial   b)   Poisson

    c)   Normal   d)   Geometric.   [   ]

x)   The truncation error in Composite Trapezoidal Rule is

    a) h^2 b) h^3

    c) h^4 d)   none of these.   [   ]

xi)   If 'a' is the actual value and 'e' is the estimated value, the formula for relative error is

    a) a/e b) $(a - e) / e$

    c) $|a - e| / a$ d) $|a - e| / e$.   [   ]

xii)   If the random variable X has p.d.f. $f(x) = \frac{1}{2}x$, $0 \le x \le 2$, find the mean value of X.

a) $\frac{4}{3}$

b) $\frac{7}{8}$

c) $\frac{5}{4}$

d) $\frac{2}{3}$.

xiii)   The relation between shift operator 'E' and forward difference operator 'Δ' is given by

a) $\Delta = 1 + E$

b) $E = 1 + \Delta$

c) $E = \Delta$

d) $E = \Delta + 2$.

xiv)   When the variance of a random variable is $\frac{2}{3}$, the Var ($3X + 5$) is

a)   6

b)   2

c)   11

d)   none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2.   Find the missing frequencies in the following frequency distribution, when it is known that A.M. = 11·09 and total frequency = 60 :

Class limits :	9·3 - 9·7	9·8 - 10·2	10·3 - 10·7	10·8 - 11·2	11·3 - 11·7	11·8 - 12·2	12·3 - 12·7	12·8 - 13·2
Frequency :	2	5	f_3	f_4	14	6	3	1·

3.   Evaluate $\displaystyle\int_0^1 (4x - 3x^2)\, dx$ taking 10 intervals, by Simpson's one-third rule.

4.   Using method of false position, find the real root of the equation

$f(x) = x^3 - 3x - 5 = 0$ upto 4 decimal places.

5.   If the third differences are constant, find y_6 if $y_0 = 9$, $y_1 = 18$, $y_2 = 20$, $y_3 = 24$.

6. Find $y^{/}$ at 0.45 from the following data :

X :	0.4	0.5	0.6	0.7	0.8
Y :	1.58	1.79	2.04	2.32	2.65

7. Fit a linear equation to the following data :

X :	2	4	6	8	10
Y :	2.3	2.6	2.9	2.10	2.12

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) There are two boxes, the first containing 3 white and 7 black balls and the second containing 7 white and 3 black balls. One box is chosen at random and from it 2 balls are drawn without replacement. Find the probability that both the balls are white. Also, given that both the balls are white, find the conditional probability that the first ball was chosen.

    b) Prove that in the limiting case, Binomial distribution tends to Poisson distribution.

    c) Prove that for two events E_1 and E_2 ,

    $$P\left(E_1 \cup E_2\right) = P\left(E_1\right) + P\left(E_2\right) - P\left(E_1 \cap E_2\right).$$ $5 + 5 + 5$

9. a) Prove that $P\left(A^c\right) = 1 - P(A)$, where A^c implies complement of the event A.

    b) Find the probability of getting exactly two even numbers when a balanced die is rolled thrice.

    c) Prove that for a Binomial distribution, mean is greater than variance. $4 + 5 + 6$

33131 (4/12)

10. a) The number of telephone calls received in 245 successive one minute intervals at an exchange are shown in the following frequency distribution :

Number of calls :	0	1	2	3	4	5	6	7	Total
Frequency :	14	21	25	43	51	40	39	12	245

Evaluate the mean and mode.

b) Compute the Standard Deviation from the following distribution of marks obtained by 90 students :

Marks :	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89	90 - 99
No. of students :	5	12	15	20	18	10	6	4

c) Find the median and median class of the following data given below :

Class boundaries :	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
Frequency :	4	11	19	14	0	2

15

11. a) Given :

X :	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

Construct the difference table and compute (i) $f(1·5)$ and (ii) $f(7·5)$.

b) Given the following data, find $f(x)$ assuming it to be a polynomial of three degree in x.

X :	0	1	2	3
$f(x)$	1	2	11	34

c) Find a positive root of $x + \ln x - 2 = 0$, by Newton-Raphson method, correct to six significant figures.

15

33131 (4/12)

12. a)   Solve the following system :

$$2x_1 - 3x_2 + 4x_3 = 8$$

$$x_1 + x_2 + 4x_3 = 15$$

$$3x_1 + 4x_2 - x_3 = 8$$

by LU-factorization, method.

b)   Solve by Euler's modified method, the following differential equation for $x = 0.02$, by taking step length $h = 0.01$,

$$\frac{dy}{dx} = x^2 + y, \quad y = 1 \text{ when } x = 0.$$

c)   Find the positive roots of the equation $x^3 - 3x + 1.06 = 0$, by method of bisection, correct to three decimal places.   15

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