

Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech/SEM-1/M-101/2009-10

2009

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following :

$$10 \times 1 = 10$$

i) The value of $\lim_{n \rightarrow \infty} \left[\frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \dots + \sqrt{n}}{n \sqrt{n}} \right]$ is

a) 0

b) 1

c) $\frac{2}{3}$

d) $\frac{3}{2}$

ii) Which of the following functions obeys Rolle's theorem in $[0, \pi]$?

a) x

b) $\sin x$

c) $\cos x$

d) $\tan x$

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iii) If C is the circle $x^2 + y^2 = 4$, then $\int_C x^2 dx$ is

- a) 0
- b) $\frac{1}{3}$
- c) 3
- d) 1.

iv) The equation $x^2 + y^2 = a^2, z = 0$ represents

- a) circle
- b) cylinder
- c) sphere
- d) right circular cylinder.

v) If θ be an angle between the vectors

$\vec{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 9\hat{j} + 6\hat{k}$, then

- a) $\theta = \cos^{-1} \left(\frac{12}{77} \right)$
- b) $\theta = \sin^{-1} \left(\frac{12}{77} \right)$
- c) $\theta = \tan^{-1} \left(\frac{12}{77} \right)$
- d) none of these.

vi) If Cauchy's mean value theorem is applicable to the function $f(x) = x$ and $g(x) = x^2$, then the value of C is

- a) $3/2$
- b) 0
- c) $1/2$
- d) $-3/2$.

vii) If $y^2 = 4ax$ (a is a real constant), then $\frac{d^2y}{dx^2} \cdot \frac{d^2x}{dy^2}$ is

- a) $\frac{2a}{y}$
- b) $\frac{2a}{y^3}$
- c) $-\frac{2a}{y^3}$
- d) $-\frac{2a}{y}$.

viii) The law of mean is given by

a) $\frac{f(b) + f(a)}{b - a} = f'(c)$

b) $\frac{f(b) + f(a)}{b + a} = f'(c)$

c) $\frac{f(b) - f(a)}{b - a} = f'(c)$

d) $\frac{f(b) - f(a)}{b - a} = f(c).$

ix) If $x = r \cos \theta$ and $y = r \sin \theta$, then the value of $\frac{\partial(r, \theta)}{\partial(x, y)}$

is

a) 0

b) r

c) $\frac{1}{r}$

d) $-r.$

x) The series $\sum_{n=1}^{\infty} \frac{2}{e^n}$ is

a) convergent

b) divergent

c) oscillatory

d) none of these.

xi) The function $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is

a) continuous and differentiable at $x = 0$

b) continuous but not differentiable at $x = 0$

c) neither continuous nor differentiable at $x = 0$

d) none of these.

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xii) If $f(x, y) = \tan(x/y)$, then $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$ is

- a) $\tan(x/y)$
- b) $\cot(x/y)$
- c) 0
- d) none of these.

xiii) The moment of inertia of a thin uniform rod of mass M and length $2a$ about an axis perpendicular to the rod at its centre is

- a) $\frac{Ma^2}{3}$
- b) $\frac{Ma^2}{2}$
- c) Ma^2
- d) $\frac{Ma^2}{4}$

xiv) The point of intersection of the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z+1}{-1}$

with the plane $x + 2y - z = 5$ is

- a) (1, 1, 1)
- b) (0, 1, 3)
- c) $\left(\frac{5}{3}, 1, \frac{-4}{3}\right)$
- d) none of these.

xv) The reduction formula of $I_n = \int_0^{\pi/2} \cos^n x \, dx$ is

- a) $I_n = \frac{n-1}{n} I_{n-1}$
- b) $I_n = \frac{n}{n-1} I_{n-1}$
- c) $I_n = \frac{n-1}{n} I_{n-2}$
- d) none of these.

GROUP - B**(Short Answer Type Questions)**Answer any *three* of the following. $3 \times 5 = 15$

2. If $y = (x^2 - 1)^n$, then show that

$$(x^2 - 1) y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$$

3. If \vec{a} , \vec{b} , \vec{c} are three vectors, then show the

$$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2, \text{ where symbols}$$

have their usual meanings.

4. Test the convergence of the series

$$1 + \frac{2^2}{3^2}x + \frac{2^2 \cdot 4^2}{3^2 \cdot 5^2}x^2 + \frac{2^2 \cdot 4^2 \cdot 6^2}{3^2 \cdot 5^2 \cdot 7^2}x^3 + \dots (x \neq 1).$$

5. A, B, C and D are points $(\alpha, 3, -1)$, $(3, 5, -3)$, $(1, 2, 3)$ and $(3, 5, 7)$ respectively. If AB is perpendicular to CD, then find the value of α .

6. If $u = \cos^{-1} \left\{ \frac{x+y}{\sqrt{x} + \sqrt{y}} \right\}$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$.

7. Verify Rolle's theorem for the function

$$f(x) = |x|, -1 \leq x \leq 1.$$

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GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Examine continuity and differentiability of $f(x)$ at $x = 0$, when $f(x) = x \sin\left(\frac{1}{x}\right)$; ($x \neq 0$) and $f(0) = 0$.

b) Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$

is not continuous at $(0, 0)$

c) Find the extrema of the function

$f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4.$ $5 + 5 + 5$

9. a) Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$ and evaluate

$\int_0^{\pi/2} \sin^5 x \, dx.$

b) If $z = f(x, y)$ where $x = e^u \cos v$, $y = e^u \sin v$ then show that

$$y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}.$$

c) Prove that the function $f(x) = |x - 1|$, $0 < x < 2$, is continuous at $x = 1$, but not differentiable there. Is it continuous and derivable at $x = 0$? $5 + 5 + 5$

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10. a) State Leibnitz's theorem for Alternating Series and test convergence of the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

b) Define absolute and conditional convergence of Series. Also show that the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ is absolutely

convergent.

6 + 9

11. a) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$.

b) Find the angles between the lines whose direction cosines are given by the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

c) Find the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

5 + 5 + 5

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12. a) Find the n -th derivative of $y = (ax + b)^m$, m is any number.

b) Test the convergence of the series

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots$$

c) Find :

$\text{div } \vec{F}$ and $\text{curl } \vec{F}$, where $\vec{F} = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$.

5 + 5 + 5

13. a) Find the whole length of the loop of the curve

$$9y^2 = (x - 2)(x - 5)^2$$

b) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x+y) dx dy$.

c) State Green's Theorem.

6 + 6 + 3