ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2009

MATHEMATICS

SEMESTER - 2

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, then A^{100} is

a) $\begin{bmatrix} 1 & 0 \\ -150 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ -50 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ -100 & 1 \end{bmatrix}$ d) None of these.

ii) The set of vectors { (2, 1, 1), (1, 2, 2), (1, 1, 1) } in R^3 is

a) linearly dependent b) linearly independent

c) basis of R^3 d) none of these.

iii) The matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is

a) an orthogonal matrix b) a symmetric matrix

c) an idempotent matrix d) a null matrix.

iv) The value of the determinant $\begin{vmatrix} 1 & 4 & 16 \\ 1^2 & 2^2 & 4^2 \\ 0 & 1 & 6 \end{vmatrix}$ is

a) 0 b) 1

c) 4 d) 22.

v) The solution of a system of n linear equations with n unknowns is unique if and only if

a) det $A = 0$ b) det $A > 0$

c) det $A < 0$ d) det $A \neq 0$,

where A is the matrix of the coefficients of the unknowns in the linear equations

vi) The eigenvalues of the matrix $\begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix}$ are

a) $-5, -3$ b) $-5, 3$

c) $3, -5$ d) $5, 3$.

vii) The general solution of $p = \log(px - y)$ where $p = \frac{dy}{dx}$ is

a) $y = cx - c$ b) $y = cx - e^c$

c) $y = c^2x - e^{-c}$ d) none of these.

viii) Which of the following is not true (the notations have their usual meaning) ?

a) $\Delta = E - 1$ b) $\Delta.\nabla = \Delta - \nabla$

c) $\frac{\Delta}{\nabla} = \Delta + \nabla$ d) $\nabla = 1 - E^{-1}$.

ix) $\Delta^2 e^x$ is equal to ($h = 1$)

a) $(e-1)^2 e^x$ b) $(e-1)e^x$.

c) $e^{2x}(e-1)$ d) e^{2x}.

x) The value of $\int_0^\infty \frac{\sin t}{t}\, dt$ is equal to

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$

c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$.

xi) If S and T are two subspaces of a vector space V, then which one of the following is a subspace of V also ?

a) $S \cup T$ b) $S \cap T$

c) $S - T$ d) $T - S$.

xii) If $\lambda^3 - 6\lambda^2 + 9\lambda - 4$ is the characteristic equation of a square matrix A, then A^{-1} is equal to

a) $A^2 - 6A + 9I$ b) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}I$

c) $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}$ d) $A^2 - 6A + 9$.

xiii) Co-factor of -3 in the determinant $\begin{vmatrix} -2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{vmatrix}$ is

a) 4 b) -4

c) 0 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If A be a skew symmetric and $(I + A)$ be a non-singular matrix, then show that $B = (I - A) (I + A)^{-1}$ is orthogonal.

3. Evaluate $L^{-1}\left\{ \dfrac{1}{(s - 1)^2 (s - 2)^3} \right\}$.

4. Solve the differential equation

$$\frac{dy}{dx} + y = y^3 (\cos x - \sin x) .$$

5. Evaluate the definite integral $\int_1^4 (x + x^3) \, dx$ by using Trapezoidal rule, taking five ordinates and calculate the error.

6. If $A (\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then show that

$$A (\theta) A (\phi) = A (\phi) . A (\theta) = A (\theta + \phi) .$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$, show that $AB = 6I$.

Utilise this result to solve the following system of equations :

$$2x + y + z = 5$$

$$x - y = 0$$

$$2x + y - z = 1$$

b) Solve : $(y - px) (p - 1) = p$ and obtain the singular solution. Here $p = \frac{dy}{dx}$.

c) Construct the interpolation polynomial for the function $y = \sin \pi x$, taking the points $x_0 = 0, x_1 = \frac{1}{6}, x_2 = \frac{1}{2}$.

Hence find $f\left(\frac{1}{3}\right)$ where $y = f(x)$.

8. a) Solve the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2 e^{3x}.$$

b) Apply suitable interpolation formula to calculate $f(9)$ correct up to two significant figures from the following data :

x :	2	4	6	8	10
$f(x)$:	5	10	17	29	49

c) Determine the conditions under which the system of equations

$$x + y + z = 1$$

$$x + 2y - z = b$$

$$5x + 7y + az = b^2$$

admits of

i) only one solution

ii) no solution

iii) many solutions.

9. a) Prove that $P^T A P$ is a symmetric or a skew-symmetric matrix according as A is symmetric or skew-symmetric.

b) Find the eigenvalues and the eigenvectors of the matrix $\begin{bmatrix} 4 & 6 \\ 2 & 9 \end{bmatrix}$.

c) Solve by Cramer's rule the following system of equations :

$$3x + y + z = 4$$

$$x - y + 2z = 6$$

$$x + 2y - z = -3.$$

10. a) What is meant by linear independence of a set of n-vectors ?

b) Solve by the method of variation of parameters the equation

$$\frac{d^2y}{dx^2} + 9y = \sec 3x .$$

c) Prove that $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$

END