ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2008

MATHEMATICS

SEMESTER - 3

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $f(z) = \bar{z}$, then $f'(0)$ is

a) 1 b) – 1

c) 0 d) does not exist.

ii) If $f(z) = u(x, y) + iv(x, y)$, then $f'(z)$ is

a) $\frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$ b) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$

c) $\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial x}$ d) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial y}$.

iii) The probability $P(a \le X \le b)$

[where $F(x)$ is distribution function of a continuous random variable X]

is defined by

a) $F(a) - F(b)$ b) $F(b) + F(a)$

c) $F(b) - F(a)$ d) $F(a)F(b)$.

iv) The number of vertices of odd degree in an undirected graph is even.

a) True b) False.

v) Let G be a graph with n vertices and e edges. G has a vertex of degree m s.t.

a) $m = \frac{2e}{n}$ b) $m \leq \frac{2e}{n}$

c) $m > \frac{2e}{n}$ d) $m \geq \frac{2e}{n}$.

vi) Given, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(AB) = \frac{1}{4}$. The value of $P\left(\bar{A}\bar{B}\right)$ is

a) $\frac{5}{12}$ b) $\frac{1}{12}$

c) $\frac{3}{4}$ d) $\frac{7}{12}$.

vii) Two variables x and y are related by $x = 2y + 5$. The median of x is 25. The median of y is

a) 9 b) 10

c) 8 d) 20.

viii) Evaluate $\int_C \frac{2}{z - \propto} dz$, where C is the circle whose equation is $|z - \propto| = \rho$

a) $4\pi i$ b) $2\pi i$

c) $\frac{\pi i}{2}$ d) πi.

ix) A purse contains 4 copper coins, 3 silver coins and an another purse contains 6 copper coins and 2 silver coins. A purse is chosen at random and a coin is taken out of it. The probability that it is a copper coin is

a) $\frac{4}{7}$ b) $\frac{3}{4}$

c) $\frac{3}{7}$ d) $\frac{37}{56}$.

x) A vertex of a tree is a cut vertex iff its degree is

a) greater than 1 b) greater than 2

c) equals to 1 d) equals to 2.

xi) If $x = 4y + 5$ and $y = kx + 4$ be two regression equations of x on y and y on x respectively, then the value of k lies in the interval

a) $[4, 5]$ b) $[0, 4]$

c) $[0, 5]$ d) none of these.

xii) A function $f(x) = x^2$, $-\pi \le x \le \pi$ is represented by a Fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$. Then the value of b_n is

a) $\frac{2\pi^2}{3}$ b) $\frac{4(-1)^n}{3}$

c) 0 d) none of these.

xiii) The period of the function $f(x) = \sin 2\pi x$ is

a) $\frac{1}{2}$ b) 1

c) 0 d) $\frac{1}{3}$.

xiv) The order of the pole $z = 0$ of function $\frac{\sin z}{z^3}$ is

a) 1 b) 2

c) 3 d) 4.

xv) Let G be a connected graph with n vertices and e edges and T be a spanning tree of G. Then T has

a) $n + 1$ branches, $e - n - 1$ edges

b) $n - 1$ branches, $e + n - 1$ edges

c) $n - 1$ branches, $e - n + 1$ edges

d) $n + 1$ branches, $e + n + 1$ edges.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Expand $f(z) = \sin z$ in a Taylor series about $z = \frac{\pi}{4}$.

3. Define component of a graph. Prove that a simple graph with n vertices and K components can have at most $\frac{1}{2}(n-K)(n-K+1)$ edges.

4. a) Suppose G is a non-directed graph with 12 edges. If G has 6 vertices each of degree 3 and the rest have degree less than 3, find the minimum number of vertices G can have.

   b) Determine the poles and residue at each pole of the function $f(z) = \cot z$.

   3 + 2

5. a) Compute Spearman's rank correlation coefficient r for the following data :

Person	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Rank in statistics	9	10	6	5	7	2	4	8	1	3
Rank in income	1	2	3	4	5	6	7	8	9	10

   b) Find the Fourier sine transform of $f(x) = \frac{1}{x}$. 3 + 2

6. State and prove Bayes theorem.

7. Find the Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$ and hence find the value of $\frac{1-1}{4} + \frac{1}{9} - \frac{1}{16} + \ldots\ldots\ldots\ldots$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) For two variables x and y the equations of two regression lines are $x + 4y + 3 = 0$ and $4x + 9y + 5 = 0$. Identify which one is 'of y on x'. Find the means of x and y. Find the correlation coefficient between x and y. Estimate the value of x when $y = 1 \cdot 5$.

b) The mean and s.d. of marks of 70 students were found to be 65 and 5·2 respectively. Later it was detected that the value 85 was recorded wrongly and therefore it was removed from the data set. Then find the mean and s.d. for the remaining 69 students.

c) Find the median from the following data :

Class :	130-134	135-139	140-144	145-149	150-154	155-159	160-164
Frequency :	5	15	28	24	17	10	1

5 + 5 + 5

9. a) If a random variable X follows normal distribution such that

$P(9{\cdot}6 \le X \le 13{\cdot}8) = 0{\cdot}7008$ and

$P(X \ge 9{\cdot}6) = 0{\cdot}8159$, where

$$\int_{-\infty}^{0\cdot9} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{t^2}{2}}\, dt = 0{\cdot}8159 \text{ and } \int_{-\infty}^{1\cdot2} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{t^2}{2}}\, dt = 0{\cdot}8849,$$

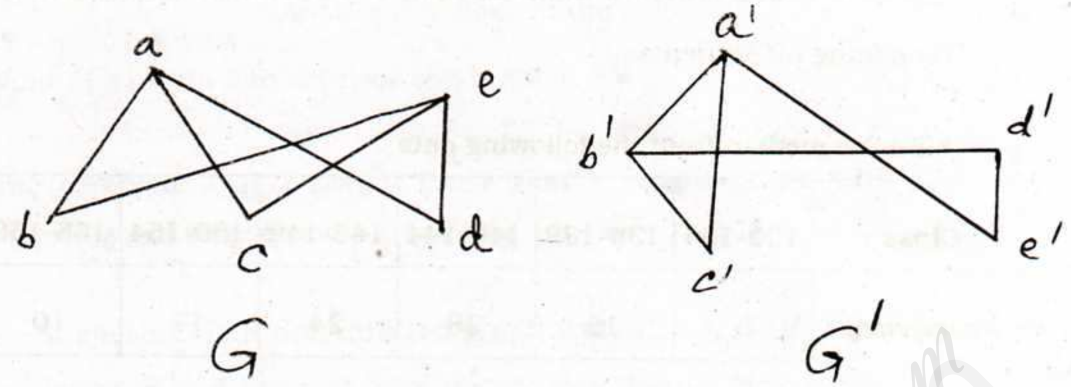
find the mean and variance of X.

b) In answering a question on a multiple choice test, a student either knows the answer or he guesses. Let P be the probability that he knows the answer and $1-p$ be the probability that he guesses. Assume that a student who guesses the answer will be correct with probability $\frac{1}{5}$. What is the conditional probability that a student knew the answer to a question given that he answered it correctly ?

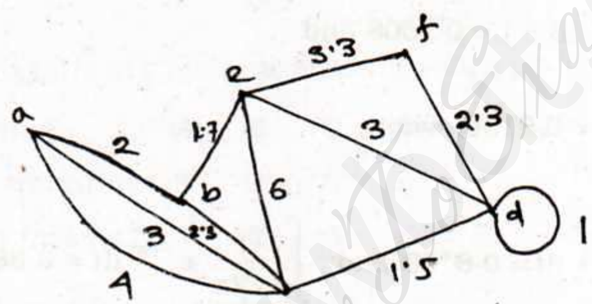
c) The overall percentage of failures in a certain examination is 40. What is the probability that out of a group of 6 candidates at least 4 passed the examination ?

5 + 5 + 5

10. a) A define isomorphism of two graphs. Examine whether the following two graphs G and G' are isomorphic. Give reasons.



b) Applying Dijkstra's method, find the shortest path between the two vertices a and f in the following graph.



8 + 7

11. a) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\, d\theta$ using Cauchy's Residue theorem.

b) Use the method of least-squares to fit a linear curve $y = a_0 + a_1 x$ to fit the data :

x_i	2	4	6	8
y_i	11	21	31	41

c) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for $0 < |z+1| < 2$.

7 + 3 + 5

12. a) Prove that the number of vertices of odd degree in a graph is always even.

b) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic in C and find a function $v(x, y)$ such that $f(z) = u + iv$ is analytic.

c) Find the Fourier Transform of the function $f(x) = 1, \; |x| \le a$

$= 0, \; |x| > a.$

Hence evaluate

$$\int_{-\infty}^{\infty} \frac{\sin sa \cos sx}{s} \, ds.$$

3 + 6 + 6

END