MECHANICAL SCIENCE—2008
SEMESTER – 2

Time : 3 Hours *Full Marks : 70*

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : 10 x 1=10

(i) Which of the following is an Intensive thermodynamic property ?
(a) Volume (b) Temperature (c) Mass (d) Energy.

(ii) For an Irreversible process, change in entropy is
(a) greater than dQ/T (b) less than dQ/T (c) zero (d) equal to dQ/T.

(iii) During throttling, which of the following quantity does not change ?
(a) Internal energy (b) Entropy (c) Pressure (d) Enthalpy.

(iv) Work done in a free expansion is
(a) Positive (b) Negative (c) Zero (d) Maximum.

(v) A cycle with constant volume heat addition and constant volume heat rejection is
(a) Otto cycle (b) Diesel cycle (c) Joule cycle (d) Rankine cycle.

(vi) Triple point of a pure substance is a point at which
(a) liquid and vapour coexist (b) solid and vapour coexist
(c) solid and liquid coexist r (d) all three phases coexist.

(vii) Bernoulli's equation deals with the conservation of
(a) Mass (b) Momentum (c) Energy (d) Work.

(viii) Continuity equation is based on the principle of conservation of
(a) Mass (b) Momentum (c) Energy (d) Entropy.

(ix) A Pitot tube is used for measuring
(a) State of fluid (b) Velocity of fluid (c) Density of fluid (d) Viscosity of fluid.

(x) Dynamic viscosity has dimensions of
(a) MLT^{-2} (b) $ML^{-2}T^{-1}$ (c) $ML^{-2}T^{-2}$ (d) $M^{-1}L^{-1}T^{-1}$

Ans. (i) b ; (ii) a ; (iii) d ; (iv) c ; (v) a ; (vi) d ; (vii) c ; (viii) b ; (ix) a ; (x) d.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. 3 x 5 = 15

2. State the first law of thermodynamics for a closed system undergoing a cycle and a process.

Ans. When a closed system undergoes any cyclic process, the cyclic integral of work is proportional to the cyclic integral of heat.

$$\oint \delta w = \oint \delta w$$

The first law of thermodynamics for a process is

$$\delta Q = dE + \delta w$$

Where δQ – heat Supplied

dE – internal energy

δw – work done

3. Explain thermodynamic equilibrium.

Ans. A System is said to exist in a state of thermodynamic equilibrium when no change in any macroscopic property is registered, if the system is isolated from its surroundings. A system will be in a state of thermodynamic equilibrium, if the conditions for the following three types of equilibrium are satisfied.

(a) Mechanical equilibrium

(b) Chemical equilibrium

(c) Thermal equilibrium

In the absence of any unbalanced force within the system itself and also between the system and the surroundings, the system is said to be in a state of mechanical equilibrium.

If there is no chemical reaction or transfer of matter from one part of the system to another, such as diffusion or solution, the system is said to exist in a state of chemical equilibrium.

When a System existing in mechanical and chemical equilibrium is separated from its surroundings by a diathermic wall and if there is no spontaneous change in any property of the system. the system is said to exist in a State of thermal equilibrium.

4. The fluid flow is given by $\overline{V} = x^2y\hat{i} + y^2z\hat{j} - (2xyz + yz^2)\hat{K}$ Show that this is a case of possible steady incompressible flow. Calculate the velocity and acceleration at (2, 1, 3).

$$\vec{v} = x^2y\hat{i} + y^2z\hat{j} - (2xyz + yz^2)\hat{k}$$

For the given fluid flow field

$u = x^2y$ $\quad\therefore \dfrac{\partial u}{\partial x} = 2xy$

$v = y^2z$ $\quad\therefore \dfrac{\partial v}{\partial y} = 2yz$

$w = -2xyz - yz^2$ $\quad\therefore \dfrac{\partial w}{\partial z} = -2xy - 2yz$

For a case of possible steady incompressible fluid flow, the continuity equation should be satisfied. i.e. $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0$

Substituting the values of $\dfrac{\partial u}{\partial x}, \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial w}{\partial z}$ we get

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=2xy+2yz-2xy-2yz=0$$

Hence the velocity field $V = x^2y\hat{i} + y^2z\hat{j} - (2xyz + yz^2)\hat{k}$
is a possible case of fluid flow.

velocity at (2,1,3)

Substituting the values x = 2, y = 1 and z = 3 in velocity field, we get

$$V = x^2y\hat{i} + y^2z\hat{j} - (2xyz + yz^2)\hat{k}$$

$$= -2^2 \times 1\,\hat{i} + 1^2 \times 3\,\hat{j} - (2\times 2\times1\times3 + 1\times 3^2)\,\hat{k}$$

$$v = 4\,\hat{i} + 3\,\hat{j} - 21\,\hat{k}$$

and resultant velocity $= \sqrt{4^2+3^2+(-21)^2} = \sqrt{466} = 21.587$ units

The acceleration components a_x, a_y and a_z for Steady flow are.

$$a_x = u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}$$

$$a_y = u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}$$

$$a_z = u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}$$

$$u = x^2y,\quad \frac{\partial u}{\partial x}=2xy, \frac{\partial u}{\partial y}=x^2 \text{ and } \frac{\partial u}{\partial z}=0$$

$$v = y^2z,\quad \frac{\partial u}{\partial x}=0, \frac{\partial v}{\partial y}=2yz, \frac{\partial v}{\partial z}=y^2$$

$$w = -2xyz - yz^2,\quad \frac{\partial w}{\partial x}=-2yz, \frac{\partial w}{\partial y}=-2xz-z^2, \frac{\partial w}{\partial z}=-2xy-2yz$$

Substituting these values in acceleration components, we get acceleration at (2, 1, 3)

$$a_x = x^2y\,(2xy) + y^2z\,(x^2) - (2xyz + y\,z^2)\,(0)$$
$$= 2x^3y^2 + x^2y^2z$$
$$= 2\times2^3\times1^2 + 2^2\times1^2\times3 = 28 \text{ units}$$

$$a_y = x^2y\,(0) + y^2z\,(2yz) - (2xyz + yz^2)\,(y^2)$$
$$= 2y^3z^2 - 2xy^3z - y^3z^2$$
$$= 2\times1^3\times3^2 - 2\times2\times1^3\times3 - 1^3\times3^2 = -3 \text{ units}$$

$$a_z = x^2y\,(-2yz) + y^2z\,(-2xz - z^2) - (2xyz + yz^2)\,(-2xy - 2yz)$$

$$= -2x^2y^2z - 2xy^2z^2 - y^2z^3 + [4x^2y^2z + 2xy^2z^2 + 4xy^2z^2 + 2y^2z^3]$$

$$= -2\times2^2\times1^2\times3 - 2\times2\times1^2\times3^2 + [4\times2^2\times1^2\times3 + 2\times2\times1^2\times4^2 + 4\times2$$

$$\times 1^2\times3^2 + 2\times1^2\times3^3]$$

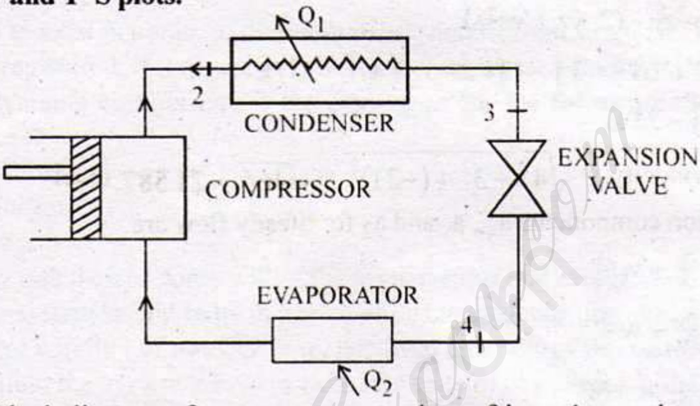
$a_z = 105$

$\therefore$ acceleration $= a_x\,\hat{i} + a_y\,\hat{j} + a_z\hat{k} = 28\,\hat{i} - 3\,\hat{j} + 105\,\hat{k}$

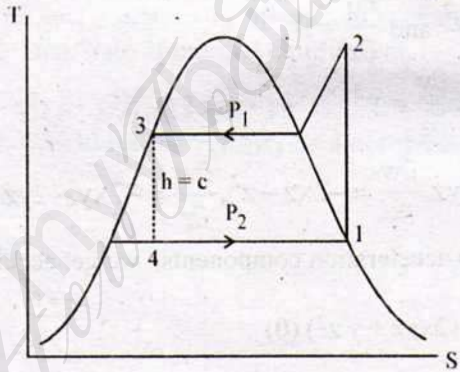
Resultant acceleration $= \sqrt{28^2 + (-3)^2 + 105^2} = \sqrt{11818} = 108.71$ units.

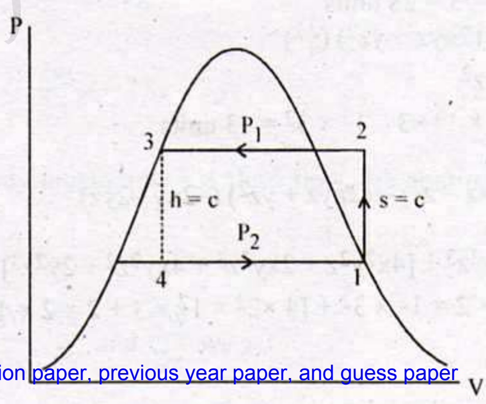
5. Draw a block diagram of vapour compression refrigeration cycle and also show the corresponding P–V and T–S plots.

Ans.



Block diagram of a vapour compression refrigeration cycle





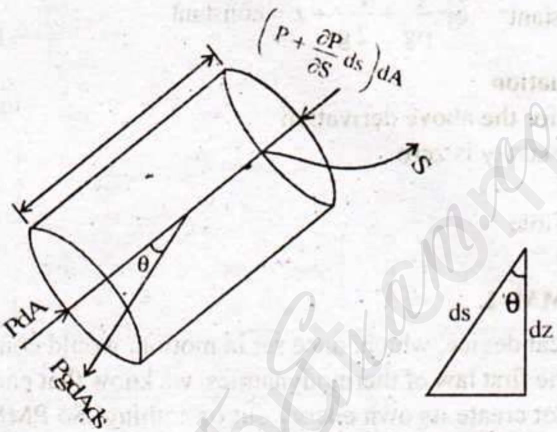
6. Derive Bernoulli's equation form first principles, stating the assumptions.

Ans. Let us consider a stream line in which flow takes place along the s–direction as shown in the figure. Let us consider a cylindrical element of this streamline having cross section dA and length ds. The forces acting on the cylindrical element are–

(i) Pressure force pdA in the direction of flow.

(ii) Pressure force $\left(P+\frac{\partial p}{s}ds\right)dA$ opposite to the direction of flow.

(iii) Weight of the element $\rho g\, dA\, ds$.



The resultant force on the fluid element in the direction of S must be equal to mass of the fluid element × acceleration in the direction.

$$\therefore\; pdA-\left(P+\frac{\partial p}{\partial s}ds\right)dA-\rho g dA ds\cos\theta=\rho dA ds\times as$$

where a_s is the acceleration in the direction of S

Now $a_s=\frac{dv}{dt}$, where v is a tunction of S and $t=\frac{\partial v}{\partial s}\cdot\frac{ds}{dt}+\frac{\partial v}{\partial t}=v\frac{\partial v}{\partial s}+\frac{\partial v}{\partial t}\left\{\because \frac{ds}{dt}=v\right\}$

If the flow is steady, $\frac{\partial v}{\partial t}=0$ $\quad\therefore a_s=v\frac{\partial v}{\partial s}$

Substituting the value of a_s in equation and simplifying the equation, we get.

$$-\frac{\partial P}{\partial s}ds\,dA-\rho g dA\,ds\cos\theta=\rho dA\,ds\times v\frac{\partial v}{\partial s}$$

Dividing by $\rho ds\, dA$ $\;-\frac{\partial P}{\rho\partial s}-g\cos\theta=\frac{v\partial v}{\partial s}$ $\quad$ or $\frac{\partial P}{\rho\partial s}+g\cos\theta+\frac{v\partial v}{\partial s}=0$

or $\frac{\partial P}{\rho \partial s} + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0$ $\left[as\cos\theta = \frac{dz}{ds} \right]$

or $\frac{\partial p}{\rho} + gdz + vdv = 0$

which is known as Euler's equation of motion. If ρ is constant i.e. for in compressible fluid, integrating Euler's equation we get,

$$\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{constant} \quad \text{or} \quad \frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

Which is Bernoullis equation

The assumptions made for the above derivation

(i) Fluid is ideal i.e viscosity is zero

(ii) Flow is steady

(iii) Flow is incompressible

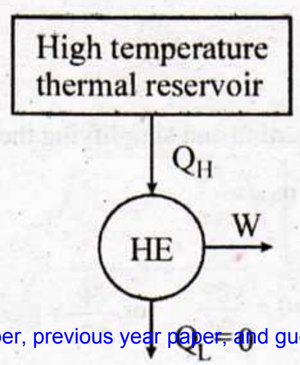
(iv) Flow is irrotational

7. Explain PMM–1 and PMM–2.

Ans. PMMI : A mechanical device, which, once set in motion, would continue to run for ever, is known as PMM 1. From the first law of thermodynamics, we know that energy transforms from one form to another. It cannot create its own energy out of nothing. So PMM 1 violates the first law and cannot exist.

PMM 1

PMM2 : It is perpetual motion machine of the second kind which produces net work in a cycle by exchanging heat with only one thermal energy reservoir and thus violates the kelvin Planck Statement.



GROUP – C

(Long Answer Type Questions)

Answer any three of the following. $3 \times 15 = 45$

8. (a) Which is a more effective way of increasing the efficiency of a Carnot engine to increase source temperatue (T_1), keeping sink temperature (T_2) constant or to decrease T_2 keeping T_1 constant.

Ans. The efficiency of a Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

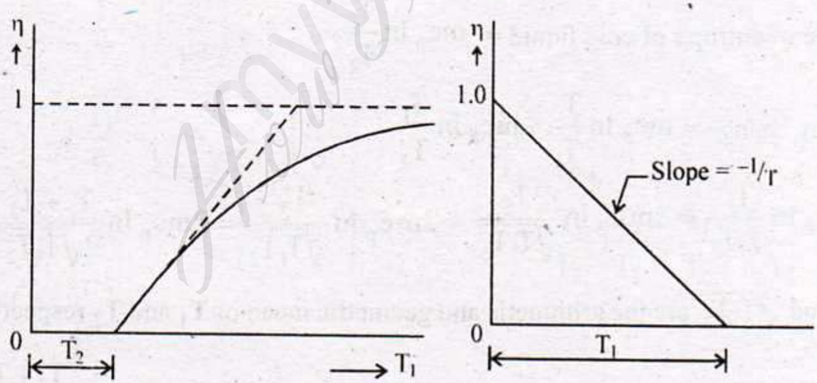
If T_2 is constant

$$\left(\frac{\partial \eta}{\partial T_1}\right)_{T2} = \frac{T_2}{T_1^2}$$

As T_1 increases, η increases and the slope $\left(\frac{\partial \eta}{\partial T_1}\right)_{T2}$ decreases. If T_1 is constant,

$$\left(\frac{\partial \eta}{\partial T_2}\right)_{T1} = -\frac{1}{T_1}$$

As T_2 increases, η increases, but the slope $\left(\frac{\partial \eta}{\partial T_2}\right)_{T1}$ remains constant.



Also $\left(\frac{\partial \eta}{\partial T_1}\right)_{T2} = \frac{T_2}{T_1^2}$ and $\left(\frac{\partial \eta}{\partial T_2}\right)_{T1} = -\frac{T_1}{T_1^2}$ since $T_1 > T_2$

$$\left(\frac{\partial \eta}{\partial T_2}\right)_{T1} > \left(\frac{\partial \eta}{\partial T_1}\right)_{T2}$$

So the more effective way to increase the efficiency is to decrease T_2

(b) State Clausius inequality.

Ans. Clausius inequality states that when a system undergoes a cyclic process, the cyclic integral of $\frac{dQ}{T}$ is always less than zero for an irreversible cycle and is equal to zero for a reversible cycle. Temperature T in the clausius inequality refers to the temperature of the thermal reservoir and not of the system.

(c) A mass of m kg of liquid (specific heat = C_p) at a temperature T_1 is mixed with an equal mass of the same liquid at a temperature T_2 ($T_1 > T_2$) and the system is thermally insulated. Show that the entropy change of the universe is given by 2m C_p $\left(\frac{T_r + T_2}{\sqrt{T_1 T_2}}\right)$ and prove that this is necessarily positive. 3 + 2+10

Ans. Let T_f be the final temperature after mixing $mc_p(T_1 - T_f) = mc_p(T_f - T_2)$

or $T_f = \frac{T_1 + T_2}{2}$

The change of entropy of hot hiquid = $mc_p \, ln \frac{T_f}{T_1}$

The change of entropy of cold liquid = $mc_p \ln \frac{T_f}{T_2}$

Net entropy change = $mc_p \ln \frac{T_f}{T_1} + mc_p \ln \frac{T_f}{T_2}$

$$= mc_p \ln \frac{T_f^2}{T_1 T_2} = 2mc_p \ln \frac{T_f}{\sqrt{T_1 T_2}} = 2mc_p \ln \frac{\frac{T_1+T_2}{2}}{\sqrt{T_1 T_2}} = 2mc_p \ln \frac{T_1 + T_2}{2\sqrt{T_1 T_2}}$$

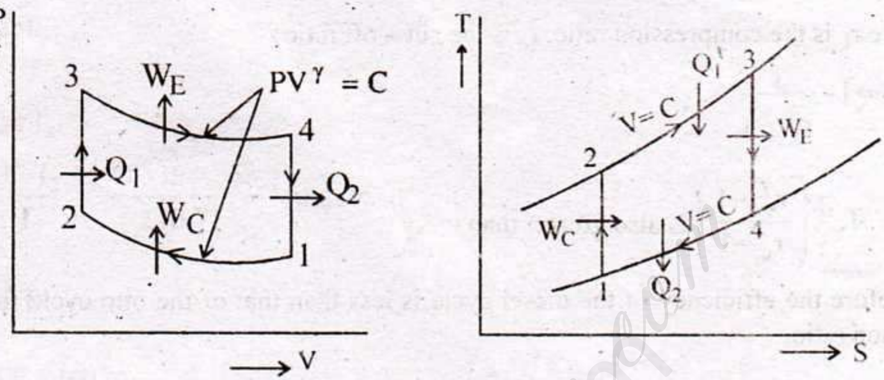
$\frac{T_1 + T_2}{2}$ and $\sqrt{T_1 T_2}$ are the arithmetic and geometric mean of T_1 and T_2 respectively. As we know that arithmetic mean is greater than that of the geometric mean, therefore $\frac{T_1 + T_2}{2}$ is greater than $\sqrt{T_1 T_2}$

$\therefore \ln \frac{T_1 + T_2}{2\sqrt{T_1 T_2}}$ is greater than zero

Net entropy change is positive.

9. (a) Derive the expression for efficiency of an Otto cycle and show the process on p–V and T–s planes.

Ans.



Heat Supplied $Q_1 = Q_{2-3} = m c_v (T_3 - T_2)$

Heat rejected $Q_2 = Q_{4-1} = m c_v (T_4 - T_1)$

Efficiency $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$

Process 1 – 2, $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$

Process 3 – 4 $\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$ $\therefore \frac{T_2}{T_1} = \frac{T_3}{T_4}$

or $\frac{T_3}{T_2} = \frac{T_4}{T_1}$ or $\frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$ $\therefore \frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1}$

$\eta = 1 - \left(\frac{v_2}{v_1}\right)^{\gamma-1}$

or $\eta_{otto} = 1 - \frac{1}{r_k^{\gamma-1}}$ $\left[\because r_k = \frac{v_1}{v_2}\right]$

Where r_k is called the compression ratio.

(b) For the same compression ratio, explain why the efficiency of Otto cycle Is greater than that of Diesel cycle.

Ans. $\eta_{diesel} = 1 - \frac{1}{\gamma} \cdot \frac{1}{r_k^{\gamma-1}} \frac{r_c^{\gamma} - 1}{r_c - 1}$

Where r_k is the compression ratio, r_c is the cut – off ratio

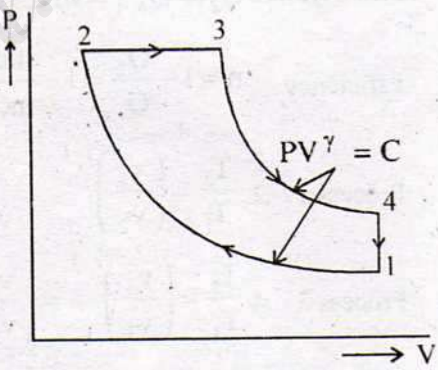
$\eta_{otto} = 1 - \frac{1}{r_k^{\gamma} - 1}$

As $r_c \rangle 1, \frac{1}{\gamma}\left(\frac{r_c^{\gamma} - 1}{r_c - 1}\right)$ is also greater than unity

Therefore the efficiency of the diesel cycle is less than that of the otto cycle for the same compression ratio.

(c) In a diesel engine the compression ratio is 13 : 1 and fuel is cut off at 8% of the stroke. Find the air standard efficiency of the engine. Take y for air = 1.4. 5+3+2+5

Ans. $r_k = \frac{v_1}{v_2} = 13$



$v_3 - v_2 = 0.08(v_1 - v_2) = 0.08(13v_2 - v_2)$
$= 0.96\, v_2$
$\therefore v_3 = 1.96\, v_2$

$r_c = \frac{v_3}{v_2} = 1.96$

$\eta_{diesel} = 1 - \frac{1}{\gamma} \cdot \frac{1}{r_k^{\gamma-1}} \frac{r_c^{\gamma} - 1}{r_c - 1}$

$= 1 - \frac{1}{1.4} \cdot \frac{1}{(13)^{0.4}} \frac{(1.96)^{1.4} - 1}{1.96 - 1} = 0.582 = 58.2\%$

10. (a) A gas occupies 0.024 m³ at 700 kPa and 95*C. It is expanded in the non–flow process according to the law pv $^{1>2}$ = constant to a pressure of 70 kPa after which it is heated at a constant pressure back to its original temperature. Sketch the process on the p–V and T–s diagrams and calculate for the whole process the work done and the heat transferred. Take C_p =1.047 and C_v = 0.775 kJ/kg K for the gas.

Ans. $P_1 V_1^{1.2} = P_2 V_2^{1.2}$

or, $700 \times 0.024^{1.2} = 70 \times v_2^{1.2}$ or, $v_2 = 0.1635$ m³

Again, $T_1 V_1^{1.2-1} = T_2 V_2^{1.2-1}$

$$\text{or } 368 \times \left(\frac{0.024}{0.1635}\right)^{0.2} = T_2$$

$$\text{or } T_2 = 250.71\text{k}$$

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

$$\text{or } V_3 = \frac{V_2 T_3}{T_2} = \frac{0.1635 \times 368}{250.71} = 0.2399\text{m}^3$$

$$PV = m R T$$

$$= m = \frac{700 \times 0.024}{0.272 \times 368} \quad \left[\because R = c_p = c_v = 1.047 - 0.775 = 0.272\right]$$

$$= 0.167 \text{ kg}$$

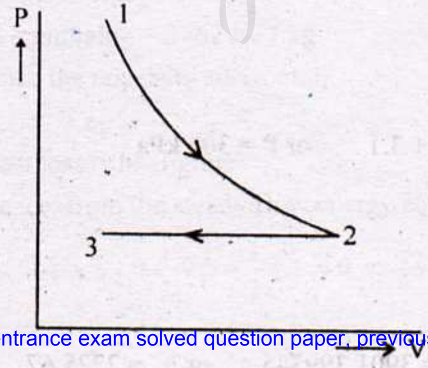
$$w_{1-2} = \frac{mR(T_1 - T_2)}{1.2 - 1} = \frac{0.167 \times 0.272 \times (368 - 250.71)}{0.2} = 26.77 \text{ KJ}$$

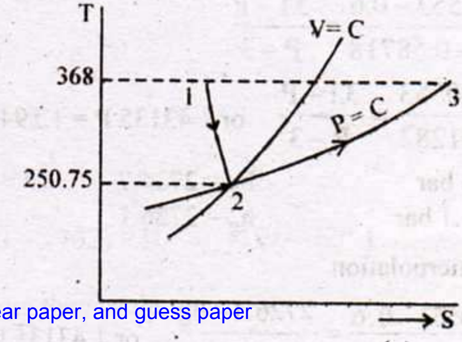
$$w_{2-3} = P_2(v_3 - v_2) = 70(0.2399 - 0.1635) = 5.35 \text{ KJ}$$

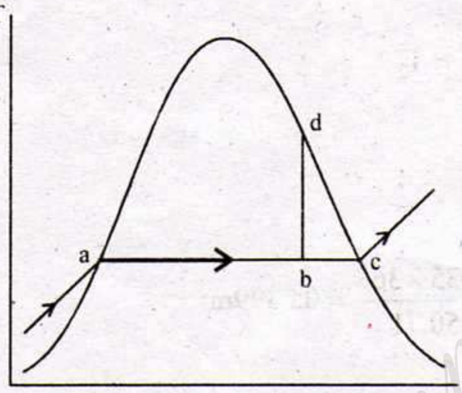
$$Q_{1-2} = (u_2 - u_1) + w_{1-2}$$

$$= mc_v(T_2 - T_1) + w_{1-2} = 0.167 \times 0.775\,(250.71 - 368) + 26.77 = 11.51 \text{ KJ}$$

$$Q_{2-3} = mc_v(T_3 - T_2) + w_{2-3} = 0.167 \times 0.775\,(368 - 250.71) + 5.35 = 20.529 \text{ KJ}$$







Specific volume = $\frac{v}{m} = \frac{3}{5} = 0.6\ m^3/kg$

Now it is wet

$v = v_g x + (1-x)v_f$ or, $0.6 = 0.88540x + (1 - x)\ 0.001061$ or, $x = 0.67727$

Now since it is closed tank volume remains constant at $0.6\ m^3/kg$

Now enthalpy at pt b is

$h = h_g x + (1-x)\ hf$

$h = 0.677 \times 2706.3 + (0.3227)\ 504.7$

$h = 1832.1651 - 162.8802$

$h = 1995.0453$

Now pressure at which the volume of the steam will be 0.6 but has to be dry saturated is required.

$P = 3$ bar $\quad V_g = 0.60553$
$P = 3.1$ bar $\quad V_g = 0.58718$

By interpolation

$$\frac{0.60553-0.6}{0.6-0.58718} = \frac{3.1-P}{P-3}$$

or $\frac{0.00553}{0.01282} = \frac{3.1-P}{P-3}$ or $1.43135\ P = 1.29407 + 3.1$ or $P = 306$ kPa

$P = 3$ bar $\quad h_g = 2724.7$
$P = 3.1$ bar $\quad h_g = 2726.1$

By interpolation

$\frac{0.60553-0.6}{0.6-0.58718} = \frac{2726.1-hg}{hg-2724.7}$ or $1.43135\ h_g = 3901.399345$ or $h_g = 2725.67$

Now

$$Q_{bd} = (v_d - u_b) + (P_d v - P_b v)$$

$$Q_{bd} = (h_d - h_b) + 0.6(P_d - P_b)$$

$$Q_{bd} = (2725.67 - 1995.0433) + 0.6(3.06 - 2) = 731.268$$

Total heat = $731.268 \times 5 = 3656.34$ KJ

Ans . The pressure is 306 kPa and the heat trans fer to the tank is 3656.34 KJ

The steady flow energy for a single stream entering and single stream leaving a control volume is

$$h_1 + \frac{v_1^2}{2} + z_1 g + \frac{dQ}{dm} = h_2 + \frac{v_2^2}{2} + z_2 g + \frac{dwx}{dm}$$

Where

h_1 , h_2 are the specific enthalpy at the inlet and the outlet respectively.

v_1 , v_2 are the velocities at the inlet and the outlet respectively.

z_1 , z_2 are the elevation above an arbitrary datum at the inlet and the outlet respectively

$\frac{dQ}{dm}$ is the heat transfer per unit mass of the fluid

$\frac{dw_x}{dm}$ is the work transfer per unit mass of the fluid

(b) A rigid closed tank of volume 3m³ contains 5 kg of wet steam at a pressure of 200 kPa. The tank is heated until the steam becomes dry saturated. Determine the pressure and the heat transfer to the tank. 8 + 7

Ans. At inlet to the nozzle

h_1 = 3000 KJ / kg = enthalpy of the flowing fluid

$\overline{v_1}$ = velocity of the fluid = 60 m/s

At the discharge end,

h_2 = enthalpy = 2762 KJ / kg

Since the nozzle is horizontal,

$z_1 = z_2$

Heat loss is negligible

(i) Hence, from the steady flow energy equation,

$$h_1 - h_2 + \frac{1}{2}\overline{v}_1^2 - \frac{1}{2}\overline{v}_2^2 = 0 \Rightarrow (3000 - 2762) \times 10^3 = \frac{1}{2}\left(\overline{v}_2^2 - 60^2\right)$$

$\Rightarrow \overline{v}_2 = 692.53$ m / s = velocity at exit from the nozzle.

(ii) A_1 = inlet area = 0.1m²

V_1 = specific volume at inlet = 0.187 m³/kg let $\dot{m}$ = mass flow rate

we know, $\dot{m} = \dfrac{A_1 \bar{v}_1}{v_1} = \dfrac{0.1 \times 60}{0.187} = 32.08 \text{ kg/s}$

(iii) V_2 = specific volume at exit = 0.498 m^3/kg

let A_2 = exit area of the nozzle

$m = \dfrac{A_2 \bar{v}_2}{v_2}$ or $A_2 = \dfrac{m v_2}{\bar{v}_2} = \dfrac{32.08 \times 0.498}{692.53}$ or $A_2 = 2.306 \times 10^{-2}\ m^2 = 0.023\ m^2$

Ans: (i) velocity at exit from the nozzle = 692.53 m / s

(ii) Mass flow rate = 32.08 kg/s

(iii) Exit area of the nozzle = 0.023 m^2

12. (a) Derive Euler's equation of motion along a streamline.

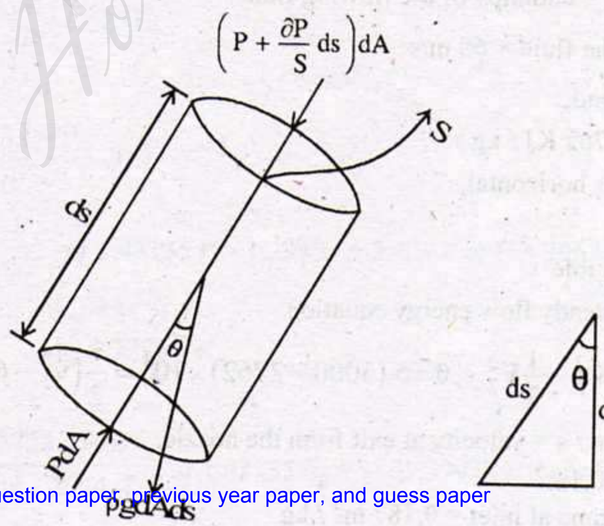
Ans. Let us consider a streamline in which flow takes place along the s direction as shown in the figure. let us consider a cylindrical element of this streamline having cross section dA and length ds. The forces acting on the cylindrical element are

(i) Pressure force pdA in the direction of flow

(ii) Pressure force $\left(P + \dfrac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.

(iii) Weight of the element ρg dA ds

Let θ be the angle between the direction of flow and the line of action of the weight of the element. The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element × acceleration in the direction S.



$$\therefore\ PdA - \left(P + \frac{\partial P}{\partial S}ds\right)dA - \rho g\, dA ds\cos\theta = \rho dA\, ds \times a_s$$

Where a_s is the acceleration in the direction of S.

Now $a_s = \dfrac{dv}{dt}$, where v is a function of s and t.

$$a_s = \frac{\partial v}{\partial S}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial S} + \frac{\partial v}{\partial t}$$

If the flow is steady, $\dfrac{\partial v}{\partial t} = 0 \qquad \therefore\ a_s = \dfrac{v\partial v}{\partial s}$

Substituting the value of a_s and simplifying the equation, we get.

$$-\frac{\partial P}{\partial s}ds dA - \rho g dA\, ds\cos\theta = \rho dA ds \times \frac{v\partial v}{\partial s}$$

Dividing by $\rho dAds$

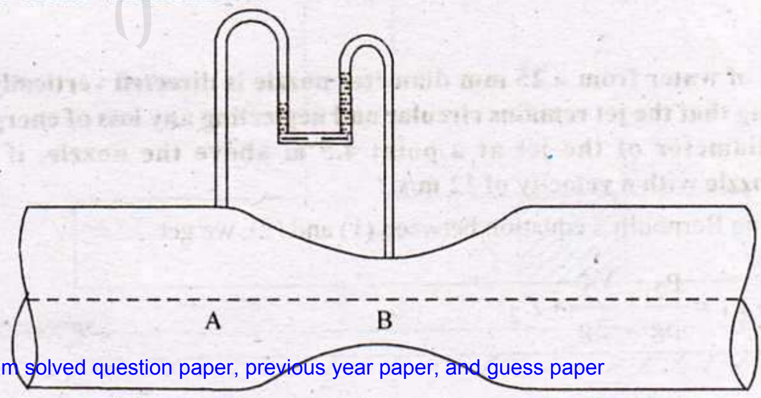
$$-\frac{\partial P}{\rho\partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$$

or $\dfrac{1}{\rho}\dfrac{\partial P}{\partial s} + g\dfrac{dz}{ds} + \dfrac{v\partial v}{\partial s} = 0 \qquad \left[\because \text{from figure } \cos\theta = \dfrac{dz}{ds}\right]$

or $\dfrac{\partial P}{\rho} + gdz + vdv = 0$

which is known as Euler's equation of motion.

(b) A venturimeter has inlet and throat diameters of 300 mm and 150 mm. Water flows through it at the rate of 0.065m ³/s and the differential gauge is deflected by 1.2 m. The specific gravity of the manometric liquid is 1.6. Determine the coefficient of discharge of the venturimeter.



Applying Bernouli's equation between A and B and considering the fluid to be inviscid, we get

$$\frac{P_A}{\rho g}+\frac{{V_A}^2}{2g}+0=\frac{P_B}{\rho g}+\frac{V_B^2}{2g}+0 \quad \text{.................(i)}$$

(the axis of the venturimeter is considered to be horizontal)

Again from continuity $V_A^2=\left(\frac{A_B}{A_A}\right)^2 V_B^2$ ....(ii)

Solving for V_B, we have

$$V_B=\sqrt{\frac{2(P_A-P_B)/\rho}{1-(A_B/A_A)^2}} \quad \text{..............(iii)}$$

The actual rate of discharge Q can be written as $Q=C_D A_B V_B$

$$=C_D A_B\sqrt{\frac{2(P_A-P_B)/\rho}{1-(A_B/A_A)^2}} \quad \text{...........(iv)}$$

Where Q is the coefficient of discharge

From the principle of hydrostatics applied to the differential gauge, we get

$$\left(\frac{P_A}{\rho g}-z\right)=\frac{P_B}{\rho g}-(z+1)+1.6\times 1 \quad \text{or} \quad \frac{P_A-P_B}{\rho g}=0.6\,\text{m}$$

Hence from eq. (iv)

$$0.065=C_D\cdot\frac{\pi}{4}\times(0.15)^2\sqrt{2\times 9.81\times 0.6/(1-\tfrac{1}{16})}$$

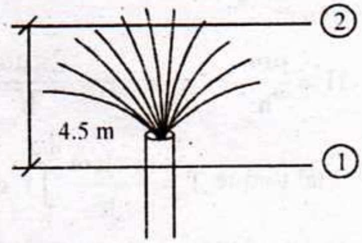
$$\text{or}\quad C_D=\frac{0.065\times 4}{\pi\times(0.15)^2\sqrt{2\times 9.81\times 0.6/(1-\tfrac{1}{16})}}=0.259$$

(c) A Jet of water from a 25 mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, what will be the diameter of the jet at a point 4.5 m above the nozzle, if the jet leaves the nozzle with a velocity of 12 m/s ? 5 + 5 + 5

Ans. Applying Bernoulli's equation between (1) and (2), we get

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

or $\frac{V_1^2}{2g}+0=\frac{P_2-P_1}{\rho g}+\frac{V_2^2}{2g}+4.5$



or $\frac{V_1^2-V_2^2}{2g}=\frac{-4.5\,\text{m of water column}}{\rho g}+4.5$

or $\frac{V_2^2-V_1^2}{2g}=\frac{2\times 4.5\times 10^3\times g}{\rho}+4.5$

or $\left(V_2^2-V_1^2\right)=(9\times 10)+4.5$

or $V_2^2=144+90+4.5=238.5$ or $\mathbf{V_2}=15.44$ m/s

Now from the equation of continuity

$A_1V_1=A_2V_2$

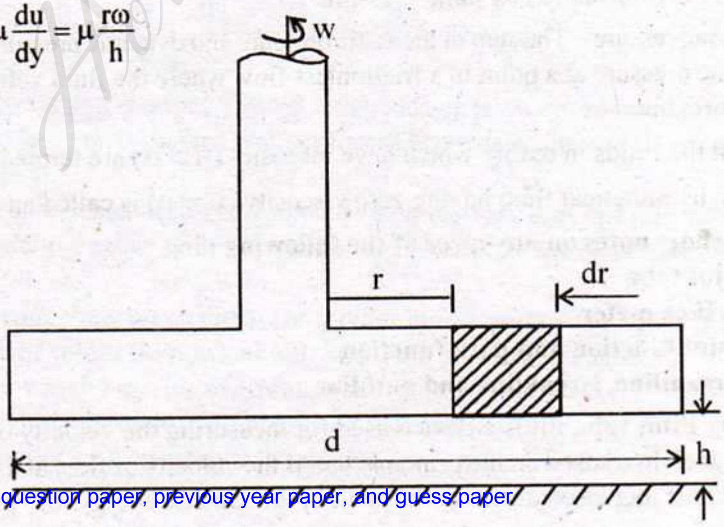
or $\frac{\pi}{4}D_1^2\times 12=\frac{\pi}{4}D_2^2\times 15.44$ or $\frac{\pi}{4}(0.025)^2\times 12=\frac{\pi}{4}\times D_2^2\times 15.44$

or $D_2=0.022\text{m}=22\text{mm}$

13. (a) A circular disk of diameter *d is* slowly rotated In a liquid of viscosity *u* at a small distance *h* from a fixed surface. Derive an expression for the torque T necessary to maintain an angular velocity ω.

Ans. (a) An element of disc is considered at a radius 'r' and having a width 'dr'. Linear velocity at this radius = rω.

Shear stress $T=\mu\frac{du}{dy}=\mu\frac{r\omega}{h}$



Assuming the gap h to be small, the velocity distribution may be assumed linear Torque dT on the element,

$$dT = \frac{\mu r \omega}{h} \times 2\pi r dr \times r = \frac{2\pi\mu\omega}{h} r^3 dr$$

$$\text{Total torque } T = \frac{2\pi\mu\omega}{h}\int_0^{d/2} r^3 dr = \frac{\pi\mu d^4\omega}{32h}$$

(b) Distinguish between the follow :

i) laminar and turbulent flow
ii) compressible and incompressible fluid
iii) static pressure and stagnation pressure
iv) viscous and inviscid fluid.

Ans. (i) Laminar flow is defined as that type of flow in which the fluid particles move along well defined paths or stream lines and all the stream lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a zigzag way. Due to the movement of fluid particles in a zigzag way, the eddies formation takes place which are responsible for high energy loss.

(ii) Compressible fluid is that type of fluid whose density changes from point to point i.e. density ρ is not constant

Incompressible fluid is that type of fluid whose density is constant.

(iii) Static pressure – It is the pressure that would be measured by an observer or pressure sensor moving with the fluid. To such an observer, the fluid appears to be static or stationary, so this pressure is often called as static pressure.

Stagnation pressure – The sum of the static pressure and dynamic pressure is called the stagnation pressure. The pressure at a point in a frictionless flow where the fluid velocity is zero is known as stagnation pressure.

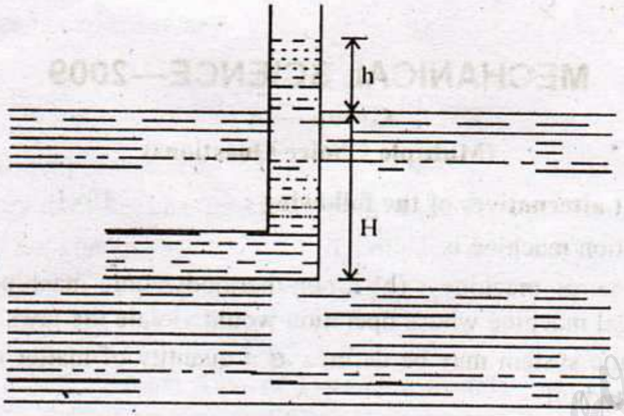
(iv) All the fluids in reality which have viscosity $(\mu > 0)$ are termed as viscous fluid.

An hypothetical fluid having zero viscosity $(\mu = 0)$ is called an inviscid fluid.

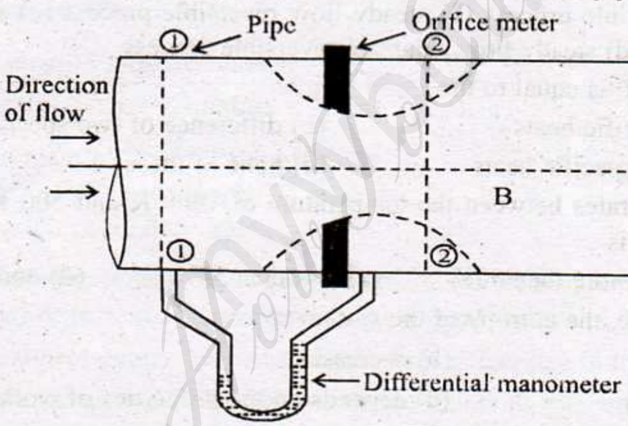
14. Write short notes on any *three* of the following :

a) Pitot tube
b) Orifice meter
c) Point function and path function
d) Streamline, streakline and pathline.

Ans. (a) Pitot tube : It is a device used for measuring the velocity of flow at any point in a pipe or a chanel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to conversion of the kinetic energy into pressure energy



Ans. (b) Orificemeter – It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of the venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.



Ans. (c) Path function is that is which a process depends on the path followed. Heat and work are path functions Point function is that in which a process depends on the end states. Temperature, pressure are point functions.

Ans. (d) Streamline at any instant can be defined as an imaginary curve or line in the flow field so that the tangent to the curve at any point represents the direction of the instantaneous velocity at that point.

Streakline – In experimental work, a die or some other indicator is of ten injected into the flow and the resulting stream of colour is known as a streakline. It gives an instantaneous picture of the positious of all particles which have passed through the point of injection.

Pathline – An individual particle of fluid does not necessarily follow a streamline but traces out a path line. In distinction to a streamline, a pathline is not an instantaneous photograph but a time exposure showing the direction taken by the same particle at successive instant of time.