



ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE - 2009
OPERATIONS RESEARCH & OPTIMIZATION TECHNIQUES
SEMESTER - 4

Time : 3 Hours]

[Full Marks : 70

Graph sheets are provided at the end of this booklet.

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) In an assignment problem the basic feasible solution for the constraint equations will consist of

- | | | |
|-------------------------|-------------------------------|----------------------|
| a) (2m + 1) variables | b) (2m - 1) variables | |
| c) 2m variables | d) 2m ² variables. | <input type="text"/> |

ii) For an LPP the dual of the dual problem is called the

- | | | |
|---------------------|------------------|----------------------|
| a) primal problem | b) dual problem | |
| c) both (a) and (b) | d) none of these | <input type="text"/> |

iii) What is the number of basic non-variables in the balanced TP with 4 rows and 5 columns ?

- | | | |
|-------|--------|----------------------|
| a) 4 | b) 5 | |
| c) 12 | d) 20. | <input type="text"/> |

iv) The full form of CPM is

- | | | |
|-----------------------------|-----------------------------|----------------------|
| a) crash project management | b) critical path management | |
| c) critical path method | d) none of these. | <input type="text"/> |

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v) A necessary and sufficient condition for a basic solution to a maximizing problem to be optimal is that (for all j)

a) $Z_j - C_j \geq 0$

b) $Z_j - C_j \leq 0$

c) $Z_j - C_j = 0$

d) $Z_j - C_j < 0$ or $Z_j - C_j > 0$.

vi) The basic feasible solutions of the system of equations

$$x_1 + x_2 + x_3 = 8$$

$$3x_1 + 2x_2 = 18$$

are

a) $(2, 6, 0), (6, 0, 2)$

b) $(1, 7, 0), (7, 1, 0)$

c) no basic solution

d) none of these.

vii) In an assignment problem, the minimum number of lines covering all zeroes in the reduced cost matrix of order n can be

a) at most n

b) $n + 1$

c) $n - 1$

d) at least n .

viii) Given a system of m simultaneous linear equations in n unknown variables ($m < n$), the number of basic variables will be

a) m

b) n

c) $n - m$

d) $m - n$.

ix) If there are n workers and n jobs, there would be

a) $n!$ solutions

b) $(n - 1)!$ solutions

c) $(n!)^n$ solutions

d) n solutions.

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- x) The point of intersection of a pure strategy game is called
- a) value of the game b) two-person game
- c) mixed strategy game d) none of these.

- xi) In an $(M/M/1) : (\infty/FIFO)$ model, the average number of customers $E(n)$ is given by
- a) ρ^n b) $\frac{\rho}{1-\rho}$
- c) $\frac{\rho^2}{1-\rho}$ d) None of these.

- xii) An assignment problem is a special type of
- a) transportation problem b) LPP
- c) inventory problem d) none of these.

- xiii) In a fair game the value of the game is
- a) 1 b) 0
- c) unbounded d) none of these.

xiv) The value of the game having the following pay-off matrix is

	B_1	B_2	B_3
A_1	10	2	3
A_2	7	6	8
A_3	0	3	1

- a) 6 b) 10
- c) 8 d) 2.

- xv) An assignment problem can be solved by
- a) Hungarian Method b) VAM
- c) Matrix Minima Method d) Dominance Principle.

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GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following.

3 × 5 = 15

2. Show that the set given by

$$X = \left\{ (x_1, x_2) : 4x_1^2 + 9x_2^2 \leq 36 \right\}$$

is a convex set.

3. Solve the following game problem :

		Player B			
		I	II	III	IV
Player A	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

4. Write the LPP in its standard form :

$$\text{Max } (Z) = x_1 - 3x_2 + 5x_3$$

subject to

$$x_1 + x_2 + x_3 \leq 7$$

$$x_1 - x_2 + x_3 \geq 2$$

$$3x_1 - x_2 + 2x_3 = -5$$

$x_1, x_2 \geq 0$ and x_3 is unrestricted.

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5. Find any one of the basic feasible solutions of the following system of equations :

$$x_1 + 2x_2 + 3x_3 = 6$$

$$2x_1 - x_2 + 4x_3 = 4$$

6. Use Big M method to maximize

$$Z = 3x_1 - x_2$$

subject to

$$2x_1 + x_2 \geq 2$$

$$x_1 + 3x_2 \leq 3$$

$$x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$

GROUP - C

(Long Answer Type Questions)

Answer any three of the following.

3 × 15 = 45

7. Use the dual simplex method to solve the following L.P.P. :

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$$\text{Maximize } Z = -2x_1 - 2x_2 - 4x_3$$

subject to the constraints

$$2x_1 + 3x_2 + 5x_3 \geq 2$$

$$3x_1 + x_2 + 7x_3 \leq 3$$

$$x_1 + 4x_2 + 6x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0.$$

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8. a) Solve the transportation problem by VAM method and checking optimality find optimum cost. 9

	D_1	D_2	D_3	
O_1	4	3	2	10
O_2	1	5	0	13
O_3	3	8	5	12
	8	5	4	

- b) Find the assignment of Machines to the Jobs, that will maximize the profit.

A B

1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

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9. A small maintenance project consists of the following jobs whose precedence relationship is given below :

Activity	Estimated Duration (weeks)		
	Optimistic	Most Likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

- a) Draw the project network. 3
- b) Find the expected duration and variance of each activity. 2
- c) Calculate the early and late occurrence for each event and the expected project length. 3

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d) Calculate the variance and standard deviations of project length. 2

e) What is the probability that the project will be completed —

- i) 4 weeks earlier than expected ?
- ii) not more than 4 weeks later than expected ?

If the project due date is 19 weeks, what is the probability of meeting the due date ?

[Given that $\phi (1.33) = 0.4082$ and $\Phi (0.666) = 0.2514$] 2 + 2 + 1

10. a) A medium project has twelve distinct activities which are to be further analyzed by using PERT. The following relevant information is also given in tabular form :

Activity	Predecessor Activity	Most optimistic time (in days)	Most likely time (in days)	Most pessimistic time (in days)
A	None	2	2	2
B	None	1	3	7
C	A	4	7	8
D	A	3	5	7
E	B	2	6	9
F	B	5	9	11
G	C, D	3	6	8
H	E	2	6	9
I	C, D	3	5	8
J	G, H	1	3	4
K	F	4	8	11
L	J, K	2	5	7

- i) Present these activities on the PERT network.
- ii) Find the expected total float for each activity.
- iii) Determine the average critical path.
- iv) Within how many days is it expected to complete the project with 99% chance ? 12

b) Find graphically the non-negative values of the variables x and y which satisfy the constraints $3x + 5y \leq 15$, $5x + 2y \leq 10$ and which maximizes the linear form $z = 6x + 3y$. 3

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11. a) Customers arrive at a one-window drive-in bank according to a Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window, including that for the serviced car can accommodate a maximum of 3 cars. Other cars can wait outside this space.

- i) What is the probability that an arriving customer can drive directly to the space in front of the window ?
- ii) What is the probability that an arriving customer will have to wait outside the indicated space ?
- iii) How long is an arriving customer expected to wait before starting service ?
- iv) How many spaces should be provided in front of the window so that all the arriving customers can wait in front of the window at least 20% of the time ?

$$4 \times 2\frac{1}{2} = 10$$

b) Find the dual of the following LPP :

Minimize $Z = 3x_1 + x_2$

subject to $x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

$x_1, x_2 \geq 0.$

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12. a) Solve the travelling salesman problem with the following cost matrix $[c_{ij}] 4 \times 4$ where, c_{ij} is the cost of travelling from city i to city j :

	1	2	3	4
1	∞	15	30	4
2	6	∞	4	1
3	10	15	∞	16
4	7	18	13	∞

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b) Solve the following Transportation Problem using the Vogel's Approximation Method :

	D_1	D_2	D_3
O_1	2	7	4
O_2	3	3	1
O_3	5	4	7
O_4	1	6	2

Also test the optimality of the solution obtained using Modified Distribution Method. 10

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END

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