# ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2008

## MATHEMATICS – I

### SEMESTER – 1

Time : 3 Hours ] [ Full Marks : 70

### GROUP – A

### ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $A = \{ 1, 2, 3, 4, 8 \}$, $B = \{ 2, 4, 6, 7 \}$, then $A \Delta B$ is

a) $\{ 2, 4 \}$

b) $\{ 1, 2, 3, 4, 6, 7, 8 \}$

c) $\phi$

d) $\{ 1, 3, 6, 7, 8 \}$. ☐

ii) $\lim_{x \to 0} (1 + x)^{1/x}$ is equal to

a) 1 b) $e$

c) 0 d) $\infty$. ☐

iii) $\frac{d}{dx} (\log_a x)$ is equal to

a) $\frac{1}{x}$ b) $\log (1/x)$

c) $(1/x) \log_a e$ d) $x \log e$. ☐

iv) If $y = \log x^2$, the value of $\frac{d^2 y}{dx^2}$ is

a) $\frac{2}{x^2}$ b) $-\frac{2}{x^2}$

c) $\frac{2}{x}$ d) $2x$. ☐

v) The matrix $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is an

a) orthogonal matrix b) idempotent matrix

c) identity matrix d) none of these.

vi) Derivative of $x^4$ with respect to $x^2$ is

a) $4x^3$ b) $2x^2$

c) $2x$ d) 4.

vii) If the roots of the equation $ax^2 + bx + c = 0$ ( $a \neq 0$ ) are real and unequal, then its discriminant $D$ satisfies

a) $D > 0$ and $D$ = a perfect square

b) $D = 0$

c) $D > 0$ and $D \neq$ a perfect square

d) $D < 0$.

viii) If $A = \{ 1, 2, 3 \}$, $B = \{ 2, 3, 6 \}$, then $A \cup B$ is

a) $\{ 1, 2, 3 \}$ b) $\{ 2, 3 \}$

c) $\{ 1, 2, 3, 6 \}$ d) none of these.

ix) If $\alpha, \beta, \gamma$ be the roots of $x^3 - 3x^2 + 6x - 2 = 0$, then $\Sigma \alpha\beta$ is

a) $-3$ b) 6

c) 2 d) none of these.

x) If $f(x) = 3 + 2x$; when $x \geq 0$

$= -3 - 2x$; when $x < 0$,

then $\lim_{x \to 0} f(x)$ is

a) 3 b) $-3$

c) 0 d) none of these.

xi) If $f(x) = \frac{x}{|x|}$ ; when $x \neq 0$

$= 1$ ; when $x = 0$, then

a) $f(x)$ is continuous at $x = 0$

b) $f(x)$ is continuous, but not differentiable at $x = 0$

c) $f(x)$ is discontinuous at $x = 0$

d) none of these. ☐

xii) The value of $\int_{-1}^{2} |x| dx$ is

a) 3 b) 5

c) 5/2 d) none of these. ☐

## GROUP – B

### ( Short Answer Type Questions )

Answer any *three* of the following. $3 \times 5 = 15$

2. If $\alpha, \beta, \gamma$ be the roots of the equation, $x^3 + px^2 + qx + r = 0$, then find the value of $\sum \alpha^3$.

3. If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$.

4. Prove that the set of even integers ( including zero ) forms an additive group.

5. Evaluate $\int_{0}^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$.

6. If $P = \begin{pmatrix} 9 & 1 \\ 4 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 5 \\ 7 & 12 \end{pmatrix}$, find the matrix $R$ so that $5P + 3Q + 2R$ is a null matrix.

## GROUP – C

### ( Long Answer Type Questions )

Answer any *three* of the following questions. $3 \times 15 = 45$

7. a) State Rolle's Theorem.

   b) Differentiate $n$ times the following equation :

   $$(1 + x^2) y_2 + (2x - 1) y_1 = 0.$$

   c) If $y = \sin\left(m \sin^{-1} x\right)$, show that

   $$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} + (m^2 - n^2) y_n = 0.$$ 4 + 5 + 6

8. a) If $p$th, $q$th and $r$th terms of an A.P. are $P$, $Q$ and $R$ respectively, show that $p(Q - R) + q(R - P) + r(P - Q) = 0$.

   b) Show that the centroid of a triangle with vertices $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

   c) Find the equation of a straight line through the point of intersection of lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ and that is perpendicular to the line $6x - 7y + 8 = 0$. 5 + 5 + 5

9. a) Show that $\cos x > 1 - \frac{x^2}{2}$ if $0 < x < \frac{\pi}{2}$.

   b) If $f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0, \end{cases}$

   then show that $f_{xy}(0, 0) = f_{yx}(0, 0)$.

   c) Evaluate $\int \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$. 4 + 6 + 5

10. a) Reduce the following equation to its canonical form and determine the nature of the conic represented by it :

    $$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0.$$

    b) Find the equation of the ellipse one of whose foci is $(-1, 1)$, eccentricity is 0·5 and the corresponding directrix is $y = x + 3$. 9 + 6

11. a) Solve the equation by Cardan's method, $2x^3 + 3x^2 + 3x + 1 = 0$.

b) Let $G = \{ a \in R / -1 < a < 1 \}$. Define a binary operation $\otimes$ on $G$ by $a \otimes b = \frac{a + b}{1 + ab}$ $\forall$ $a, b \in G$. Show that $( G, \otimes )$ is a group.

c) Find the nature of the roots $x^4 + qx^2 + rx - s = 0$ by Descartes' rule of signs ( where $q, r, s$, being positive ). 15

12. a) If by a trnasformation of one rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$,

prove that $a^2 + b^2 = a'^2 + b'^2$,

b) Show that $\int_0^{\infty} \frac{dx}{(x+1)(x+2)} = \log 2$.

c) Use the method of integration to evaluate $\lim_{n \to \infty} \frac{1^k + 2^k + \ldots + n^k}{n^{k+1}}$ ; $k > 0$.

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END