

Name :

Roll No. :

Invigilator's Signature :

CS/BCA/SEM-2/BM-201/2010
2010
MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following : 10 × 1 = 10

- i) The basis of a vector space contains
- a) linearly independent set of vectors
 - b) linearly dependent set of vectors
 - c) scalars only
 - d) none of these.

ii) The solution of $\frac{d^2y}{dx^2} = 0$ is

- a) $y = e^x$
- b) $y = 0$
- c) $y = \sin x$
- d) $y = \log_e x$.

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iii) If $f(3, 1) = x(1, 2) + y(0, 3)$ then the values of x and y are respectively

- a) $(3, -5)$ b) $(3, 1)$
c) $(3, -5/3)$ d) $(3, -5/2)$.

iv) $\lim_{n \rightarrow \infty} (3n + 1) / (2n - 3)$ is

- a) $\frac{1}{2}$ b) $\frac{3}{2}$
c) 1 d) $-\frac{1}{3}$.

v) The value of $(1/D^2)(x^3)$ is

- a) x^5 b) $\frac{1}{20}$
c) 20 d) $\frac{1}{20}x^5$.

vi) $\sum 1/n^p$ is divergent if

- a) $p \leq 1$ b) $p > 1$
c) $p < 1$ d) $p = 1$.

vii) If $P = \{2, 4, 6, 7, 8, 9\}$, $Q = \{1, 2, 6, 9\}$, then $P - Q$ is

- a) $\{4, 7, 8\}$
b) $\{4, 6, 8, 9\}$
c) $\{1\}$
d) $\{2, 4, 6, 7, 8, 9\}$.

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viii) $\frac{1}{(D-2)(D-3)}e^{2x}$ is

- a) $-e^{2x}$
- b) xe^{2x}
- c) $-xe^{2x}$
- d) $-xe^{3x}$

ix) Integrating factor of $\frac{dy}{dx} + y = x$ is

- a) e^{-x}
- b) e^x
- c) x^2
- d) none of these.

x) The differential equation $\left(\frac{dy}{dx}\right)^2 + ay^2 = x$ is

- a) linear of degree 2
- b) non-linear of order one and degree 4
- c) non-linear of order one and degree 2
- d) none of these.

xi) If vectors $(a, 0, 1), (0, 1, 0), (1, a, 1)$ of a vector space \mathbb{R}^3 over \mathbb{R} be linearly dependent, then the value of a is

- a) 2
- b) 3
- c) 1
- d) none of these.

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xii) Auxiliary equation of the differential equation $\frac{d^2y}{dx^2} + 4y = \sin x$ is

- a) $y = \cos 2x + \sin 2x$
- b) $y = c_1 \cos 2x + c_2 \sin 2x$
- c) $y = c_1 \cos x + \sin 2x$
- d) none of these.

xiii) The general solution of $\log \frac{dy}{dx} = x - y$ is

- a) $e^y - e^x = c$
- b) $e^x + e^y = c$
- c) $e^{x+y} = c$
- d) $e^{x-y} = c$.

xiv) If S and T be two subspaces of a vector space V , then which of the following is also a subspace of V ?

- a) $S \cup T$
- b) $S - T$
- c) $T - S$
- d) $S \cap T$.

GROUP - B

(Short Answer Type Questions)

Answer any three of the following. $3 \times 5 = 15$

- 2. Show that the sequence $\{2 + (-1)^n 1/n\}$ is convergent.
- 3. Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 + e^{3x}$

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4. Find the value of x for which the vectors $(1, 2, 1)$, $(x, 3, 1)$ and $(2, x, 0)$ become linearly independent.
5. Find the value of the limit $\lim_{n \rightarrow \infty} (4n^3 + 6n - 7) / (n^3 - 2n^2 + 1)$.
6. Find a basis and the dimension of $S \cap T$, where S and T are subspaces of R^3 defined by

$$S = \{ (x, y, z) \in R^3 : 2x + y + 3z = 0 \}$$

$$\text{and } T = \{ (x, y, z) \in R^3 : x + 2y + z = 0 \}$$

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that $\left\{ \frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \dots + \frac{1}{\sqrt{n^2 + n}} \right\}$ is convergent and converges to 1.
- b) Show that the sequence $\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots$ converges to 2.

8 + 7

8. Solve the following equations :

3 × 5

a) $(D^2 - 2D + 1)y = x \sin x$

b) $\frac{d^2y}{dx^2} + \frac{1}{x} \cdot \frac{dy}{dx} = \frac{12 \log x}{x^2}$

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c) $3\frac{dy}{dx} + 2\frac{y}{x+1} = \frac{x^3}{y^2}$

9. a) Prove that a subset S of a vector space V over R is a subspace if and only if $\alpha x + \beta y \in S$ for all $\alpha, \beta \in R$ and $x, y \in S$.

b) Prove that the vectors $\{ (1, 2, 2), (2, 1, 2), (2, 2, 1) \}$ are linearly independent in R^3 .

c) Find the basis and the dimension of the subspace W of R^3 where

$W = \{ (x, y, z) \in R^3 : x + y + z = 0 \}$ 5 + 5 + 5

10. a) Solve $(px - y)(py + x) = a^2p$, by using the substitution $x^2 = u, y^2 = v$; where $p = \frac{dy}{dx}$.

b) Obtain the general solution and singular solution of the equation $y = px + \sqrt{a^2p^2 + b^2}$. 7 + 8

11. a) Define basis of a vector space.

b) Show that the vectors $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ form a basis for R^3 . Express $(1, 0, 0)$ as a linear combination of α_1, α_2 and α_3 .

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- c) Find the matrix of the linear transformation T on $V_3(\mathbb{R})$ defined as

$T(a, b, c) = (.2b + c, a - 4b, 3a)$ with respect to the ordered basis B where

$$B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}. \quad 3 + 6 + 6$$

12. a) Prove that the sequence $\{a_n\}$ is monotonically increasing and bounded when

$$a_n = (3n + 1)/(n + 2)$$

- b) State D' Alembert's Ratio Test.

- c) If α, β, γ form a basis of a vector space V , then prove that $\alpha + \gamma, 2\alpha + 3\beta + 4\gamma$ and $\alpha + 2\beta + 3\gamma$ also form a basis of the vector space V .

8 + 2 + 5