PGDOI4RC4

S.Y. Diploma in Operation Research. May. 2008.
Paper II. Advance Operation Research I (For Finance. stream, marketing stream & production stream)

Mha 1 -07 595

Con. 2175 & (a)(b)-08. **BB-8593 to 8595**

(3 Hours) **[Total Marks : 100**

N.B. :— (1) Answer any **five** questions.
(2) **Figures** to the **right** indicate **full** marks.
(3) Use of **Statistical Tables** and **Non-Programmable calculator** is **permitted.**
(4) Answers must be brief and **to the points.**
(5) Intermediate **explanations** and **calculations** must be given.
(6) Assumptions, **wherever necessary,** must be clearly stated.

1. (a) Solve the following LPP using Branch and Bound Algorithm :— **10**

Maximize $Z = x_1 + x_2$
subject to $2x_1 + 5x_2 \leq 16$, $6x_1 + 5x_2 \leq 30$,
x_1 and x_2 are non-negative integers.

(b) Solve the following LPP by Dual Simplex Method **10**

Minimize $Z = 4x_1 + 3x_2$
Subject to $x_1 + 3x_2 \geq 30$
$-x_1 + 2x_2 \geq 20$
$x_1, x_2, \geq 0$

2. (a) Solve the following parametric programming problem :— **10**

Maximize $z = 3x_1 + 2x_2 + 5x_3$
subject to $x_1 + 2x_2 + x_3 \leq 30 - 7t$
$3x_1 + 2x_3 \leq 60 + 2t$
$x_1 + 4x_2 \leq 40 - t$
$x_1, x_2, x_3, t \geq 0$

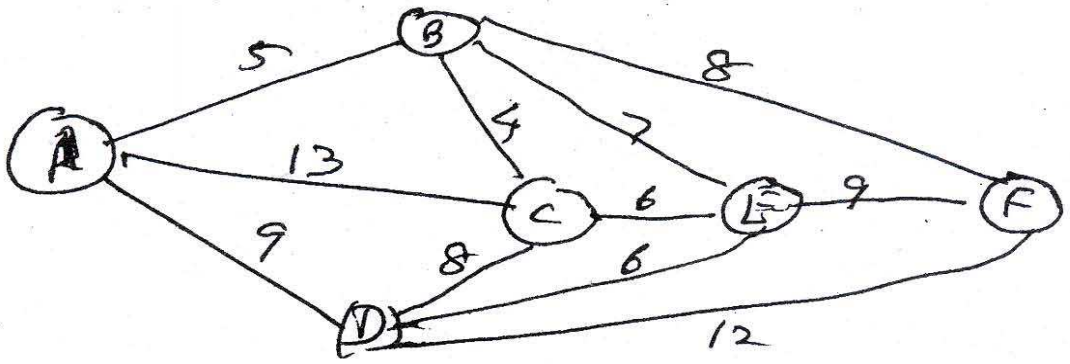
(b) Write down the Dual Problem for the following LPP : **10**

Minimize $Z = 5x_1 + 3x_2 - 4x_3$
subject to $x_1 + x_2, + x_3 \geq 5$
$2x_1 - 5x_2 + x_3 = 6$
$x_1, x_2 \geq 0$ and x_3 is unrestricted

3. (a) Solve the following problem by Revised Simplex method : **10**

Minimize $Z = 42x_1 + 45x_2 + 40x_3$
Subject to $x_1 + 3x_2 + 4x_3 \geq 3$
$2x_1 + 2x_2 + 4x_3 \geq 2$
$x_1 + 2x_2 + 4x_3 \geq 5$,
$x_1, x_2, x_3 \geq 0$

(b) The following information gives distances in kms between 6 cities A, B, C, D, E, F. **10**



Draw the Spanning Tree.

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4. Determine an optimum solution for the following LPP : 20

Maximize $Z = 25x_1 + 6x_2 + 8x_3$

subject to $2x_1 + 4x_2 - 3x_3 \leq 30$

$x_1 + 3x_2 + x_3 \leq 25$

$x_1, x_2, x_3 \geq 0$.

(a) Find the change in the optimum solution if c_1 changes from 25 to 30

(b) If a new contraint, $x_1 + 2x_2 - x_3 \leq 30$ is added find the new optimum solution.

(c) If the RHS of the constraints changes from $\begin{pmatrix}30\\25\end{pmatrix}$ to $\begin{pmatrix}40\\20\end{pmatrix}$, what will be the changed optimum solution ?

5. Maximize $Z = 30x_1 + 20x_2 + 25x_3$ 20

subject to $3x_1 + 2x_2 + x_3 \leq 120$ (supply of Resource I)

$5x_1 + 3x_2 + 2x_3 \leq 100$ (supply of Resource 2)

Further it was possible to acquire more units of Resources 1 and 2. Management would therefore prefer to buy excess units of Resource 2.

The goods were set as to minimize

(a) deviation from a target profit of Rs. 2500

(b) additional units of Resource 2 and

(c) additional units of Resource 1.

Formulate the problem as a Goal Programming problem and solve it.

6. (a) Find the maximum flow (units) of an item that can be transported through a network with the paths listed below from source 1 to sink 5. 10

The paths and their capacities in units (x) are listed below :

Paths	1–2	1–3	2–4	2–5	3–4	3–5	2–3
x	9	12	4	11	6	10	4

Indicate the paths fully utilizing their capacities available.

(b) The following table gives activities, durations, in days and manpower requirements for a certain project : 10

Activity	1–2	1–3	2–4	3–4	4–5
Duration	3	2	1	2	2
Manpower	4	3	2	5	4

Level out the resources without increasing the project duration.

7. (a) The following table gives optimistic (a) most likely (m), and passimistic (c) durations, in days, for activities of a certain project : 10

Activity	1–2	1–3	1–4	2–3	2–5	3–4	4–5
a	1	3	2	2	3	1	1
m	3	4	5	6	4	3	2
b	5	5	8	10	5	5	3

(i) Find the mean and variance of each activity duration.

(ii) What is the probability that the project will be completed in 5 days more than the expected duration of the project.

(b) The table below gives estimates of Normal Durations (ND), Crash Durations (CD), Normal Cost (NC) and Crash Cost (CC) for each of the project activities. **10**

Activity	1–2	1–3	2–3	3–4	4–5
ND, days	4	5	3	4	6
CD, days	2	3	2	1	2
NC, Rs. 100	50	150	80	110	200
CC, Rs. 100	100	170	110	260	275

Indirect Cost is Rs. 60 per day.
Determine :
(i) the minimum cost project duration
(ii) the minimum project duration and the corresponding cost.

8. (a) For the following project, determine EST, LFT, EFT, LST, Critical path and its duration : **10**

Activity	1–2	1–3	2–5	3–4	4–5
Duration (days)	3	2	5	6	4

(b) Explain the following terms with illustration : **10**
(i) Total Float
(ii) Gomery's Cutting Plane algorithm
(iii) Aternative optimum, No feasible solution, unbounded solution and degeneracy in LPP.

Con. 2175(a)-08. **BB-8594**

(3 Hours) **[Total Marks : 100**

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(3) Use of **Statistical Tables** and **Non-Programmable calculator** is **permitted.**
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(5) Intermadiate **explanations** and **calculations** must be given.
(6) Assumptions, **wherever necessary,** must be clearly stated.

1. (a) Solve the following LPP : **10**

Maximize $z = 30x_1 + 16x_2 + 25x_3$
subject to $8x_1 + 4x_2 + 5x_3 \leq 1000$
$5x_1 + 3x_2 + 3x_3 \leq 650$
$3x_1 + 2x_2 + 3x_3 \leq 420, \quad x_1, x_2, x_3 \geq 0$

(b) Write down the Dual of the following LPP : **10**

Minimize $z = 2x_2 + 5x_3$
subject to $x_1 + x_2 \geq 2,$
$2x_1 + x_2 + 6x_3 \leq 6$
$x_1 - x_2 + 3x_3 = 4, \quad x_1, x_2, x_3 \geq 0$

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2. A farmer has 300 acres of land for cultivating two crops P_1 and P_2. The cost of cultivating P_1 is Rs. 500 per acre, whereas that of P_2 is Rs. 700 per acre. Budger fir cultivation available is Rs. 80,000. **20**

Each acre of P_1 requires 20 hours of labour and for P_2 25 hours of labour per acre. Maximum labour hours available are 4000 hours.

The farmer wants to cultivate at least 80 acres of P_1. He wants to make a profit of Rs. 200 of P_1 and Rs. 300 of B. He wants to maximize the total profit.

Formulate this problem as a LPP and using Simplex method, find the optimum solution.

3. (a) State the advantages of Revised Simplex Method over Standard Simplex Method. **10**

(b) Solve the following LPP using Revised Simplex Method :— **10**

Maximize $Z = 3x_1 + 5x_2 + 8x_3$

subject to $5x_1 + 3x_2 + 4x_3 \leq 20$

$2x_1 + 4x_3 + 5x_3 \leq 30$

$x_1, x_2, x_3 \geq 0$

4. (a) Solve the following LPP by Dual Simplex Method :— **10**

Minimize $Z = 3x_1 + 3x_2 + x_3$

subject to $2x_1 + x_2 + x_3 \geq 6$

$x_1 + x_2 + 2x_3 \leq 8, \quad x_1, x_2, x_3 \geq 0$

(b) For the following LPP **10**

Minimize $Z = 190x_1 + 140x_2 + 200x_3$

subject to $2x_1 + x_2 + x_3 \geq 70$

$x_1 + x_2 + 2x_3 \geq 80, \quad x_1, x_2, x_3 \geq 0$

Write down the Dual Problem and solve it. Hence find the optimum solution for the above given problem.

5. Find the optimum solution for the following LPP :

Maximize $Z = 2x_1 + 3x_2$

subject to $3x_1 + x_2 \leq 22, \quad x_1 + 3x_2 \leq 26$

$x_1, x_2 \geq 0$

Find the effect on the optimum solution if :—

(i) C_1 changes from 2 to 4 **5**

(ii) b_2 changes from 24 to 20 **5**

(iii) coefficients of x_1 in the constraints change from $\begin{pmatrix}3\\1\end{pmatrix}$ to $\begin{pmatrix}2\\2\end{pmatrix}$, **5**

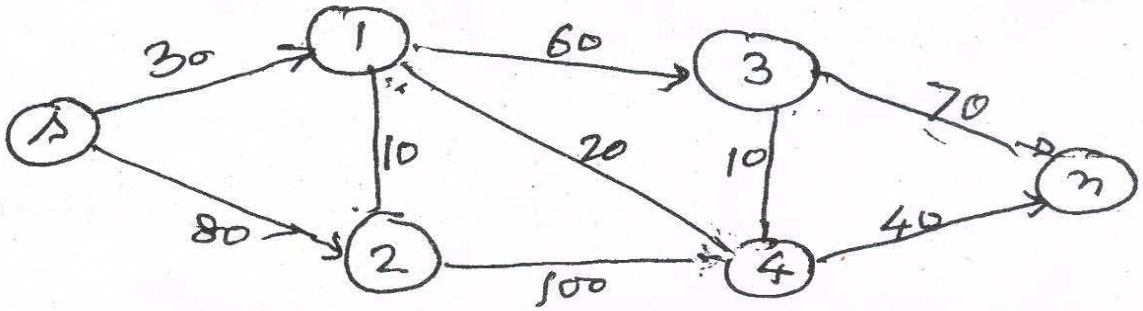
(iv) a new constraint $2x_1 + 2x_2 \leq 25$ is added. **5**

6. (a) The following table indicates duration, in days, and number of workers required for each activity of the project : **10**

Activity	1–2	1–3	2–4	3–5	4–5
Duration	2	3	1	2	3
No. of Workers	4	2	3	4	2

Level out the manpower without increasing the project duration.

(b) Consider a street network as shown below :— **10**



Numbers on the arcs indicate traffic flow capacities. The problem is to place one-way signals on the streets, not already oriented, so as to maximize the trafic flow from point s to point n.

7. (a) The following table shows for a certain project, Normal Duration (ND), and Crash Duration (CD), in days, Normal Cost (NC) and Crash Cost (CC), in Rs. for activities of a certain project. **10**

Activity	1–2	1–3	2–4	2–5	3–4	4–5
ND	5	3	4	12	7	5
CD	4	2	3	7	3	3
NC	40	60	15	40	50	25
CC	90	170	35	190	90	50

Indirect Cost is Rs. 30 per day.
Determine :

(i) Minimum cost duration of the project.
(ii) Minimum Duration cost for the entire project.

(b) Optimistic (a), most likely (m) and pessimestic (b) durations for a certain project are given below : **10**

Activity	1–2	1–3	2–4	3–4	4–5
a	3	6	4	9	5
m	4	9	7	12	8
b	5	12	10	15	11

(i) Determine the expected duration and variance for each activity.
(ii) What is the probability that the project will be completed in more them 4 days after its expected duration ?

8. (a) Consider the following pay-off matrix, with Player A as the maximising player : **10**

		B		
		B_1	B_2	B_3
A	A_1	3	5	8
	A_2	–3	3	8
	A_3	4	3	6

Obtain the optimum solution for the above game problem.

(b) For a washing powder manufacturing factory, frequency distribution of contribution (x) (= sales price – variable cost) per unit, annual demand (y) and requirement of investment (z) hence found as follows : **10**

x	3	5	7	9
Prob.	·1	·3	·4	·2

y	20	25	30	35
Prob.	·1	·2	·6	·1

z	175	200	300
Prob.	·3	·5	·2

Using Mont Carlo Simulation of 5 runs, estimate the percentage of return on investment.
Use the following random numbers :

3025 1348 0752 3148 1234 5897 6325
1485 2008 3529 7784 3948 2145 6974

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(5) Intermadiate **explanations** and **calculations** must be given.
(6) Assumptions, **wherever, necessary,** must be clearly stated.

1. (a) Solve the following LPP :— **10**

Maximize $Z = 3x_1 + 5x_2 + x_3$
subject to $2x_1 + 4x_2 + 3x_3 \leq 40$
$x_1 + x_2 \leq 10$
$2x_2 + x_3 \leq 12,\ x_1, x_2, x_3 \geq 0$

(b) Write down the dual problem for the following LPP : **10**

Maximize $Z = -5x_1 - x_2 - 2x_3$
subject to $2x_1 + 3x_2 + x_3 \geq 10$
$x_1 - 2x_2 + 4x_3 = 20$
$x_1, x_2 \geq 0$ and x_3 unrestricted.

2. A manufacturer produces 3 products A, B and C by using 2 types of machines Lathes (M_1) and Milling (M_2). The necessary information is given in the following table : **20**

Machine	Time, minute, per unit A	B	C	Time available per day in min.
M_1	7	10	4	1000
M_2	3	40	1	600
Profit, Rs. per unit	45	100	30	

(a) Formulate the problem as an LPP and obtain the optimum solution.
(b) Find the effect of changing C_1 from Rs. 45 to Rs. 30 and C_2 from Rs. 100 to Rs. 40.
(c) Find the effect of changing the total time per day available on M_1 and M_2 to 800 and 500 minutes respectively.
(d) A new product D requires 15 minutes on M_1 and 10 minutes on M_2 per unit. Will the product D will be produced if its profit, per unit, is Rs. 50 ?
(e) Each product requires 2, 5 and 3 minutes per unit of time on a third machine, M_3 find the new optimum solution, if available time, per day, on M_3 is 800 minutes.

3. (a) Solve the following LPP using graphical method : **10**

Maximize $Z = 2x_4 - x_2$
subejct to $3x_1 + x_2 = 3,\ 4x_1 + 3x_2 \geq 6,$
$x_1 + 2x_2 \leq 4,\ x_1, x_2, \geq 0$

(b) Obtain an integer solution for the following LPP : **10**

Maximize $Z = 7x_1 + 9x_2$
subject to $-x_1 + 3x_2 \leq 6$
$7x_1 + x_2 \leq 35,\ x_1, x_2 \geq 0$ and integers.

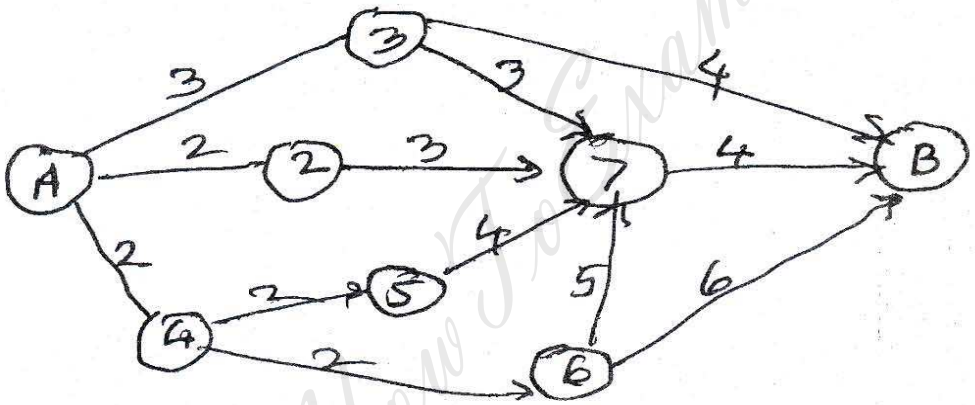
4. Solve the following Parametric Linear Programming Problem : **20**

Maximize $Z = (30 + 3t)\ x_1 + (40 - 2t)\ x_2 + 20x_3$
subject to $3x_1 + 3x_2 + x_3 \leq 20$
$2x_1 + 4x_2\ 3x_3 \leq 30$
$6x_1 + x_2 + x_3 \leq 20$
$x_1, x_2, x_3 \geq 0,\ t \geq 0$

5. Maximize $Z = 40x_1 + 45x_2 + 24x_3$ **20**
subject to $2x_1 + 5x_2 + x_3 \leq 100$ (Availability of Steel)
$3x_1 + 3x_2 + 2x_3 \leq 120$ (Availability of Rubber)
$x_1, x_2, x_3 \geq 0$
Further it was possible to acquire more steel and rubber at the same original cost. But the location of steel supplier was closer. Management therefore, would prefer to buy excess of steel before buying excess rubber. The goals were set as to minimize :
(a) deviation from a target profit of Rs. 2250
(b) additional rubber purchase and
(c) additional steel purchase.
Formulate the problem as a Goal Programming Problem and find the optimum solution.

6. (a) Solve the following LPP by Dual Simplex Method : **10**
Minimize $Z = 4x_1 + 3x_2 - x_3$
subject to $x_1 + 3x_2 - 2x_3 \geq 20$
$-x_1 + 2x_2 + 4x_3 \geq 25$, $x_1, x_2, x_3 \geq 0$.

(b) For the following network, determine the **10**
(i) Shortest route from A to B and
(ii) Critical Path
where A and B are source and sink nodes respectively.



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7. (a) For the following net-work **10**

Activity	1–2	1–3	1–4	2–4	2–5	3–4	4–5
Duration (days)	3	2	6	5	7	2	4

determine the critical path, project duration and Total Float for each activity.

(b) The following table gives activities, their duration and manpower requirements for a certain project.

Activity	1–2	1–3	2–4	3–4	3–5	4–5
Duration (days)	3	2	6	7	5	4
No. of men	2	3	1	3	2	3

Level out the man power without increasing the project duration.

8. (a) A maintenance project consists of the jobs shown below. Normal Duration (ND), Crash Duration (CD) in days, Normal Cost (NC), Crash Cost (CC) in Rupees, for each job are also given below : **10**

Activity	ND	CD	NC	CC
1–2	5	2	100	250
1–3	2	1	50	80
2–4	5	3	150	190
3–4	3	1	80	120
4–5	2	1	60	70

Determine :

(i) the project duration when the total cost is minimum, and

(ii) the total project cost when the project duration is minimum.

(b) Duration of activities are uncertain. The following table gives optimistic, most likely and pessimistic duration of each activity, in days : **10**

Activity	1–2	1–3	2–3	2–4	3–5	4–5
Optimistic duration	5	15	16	8	7	3
Most likely duration	9	20	21	12	10	6
Pessimistic duration	13	25	26	16	13	9

What is the probability that the project will be completed in 4 days later than the expected project duration ?

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