(2) Attempt any **four** questions out of the remaining **six** questions.
(3) Assume any **suitable** data, wherever **required**.
(4) **Figures** to the **right** indicate marks.
(5) Answers to the subquestions should be grouped **together** and written **together**.

M.E. (Mech) Sem V PTDC mc Design Finite Element Analysis 1776/08

1. Using Rayleigh Ritz method, over **general element**, solve the following differential equation to get Element Matrix Equation. 20

$$\frac{d}{dx}\left(KA\frac{d\theta}{dx}\right) - hp\theta = 0.$$

where θ = Temperature different = $T_x - T_\infty$
T_x = Temperature at a distance 'x' from Left Hand End, °C
T_∞ = Surrounding Temperature, °C
K = Thermal conducting, w/m, °C
A = Cross-sectional Area, m^2
h = Convective Heat Transfer Co-efficient, w/m^2 °C
P = Perimeter, M

Write all the steps carefully.
Boundary conditions : (i) $\theta(0) = \theta_0$
(ii) No heat transfer at Right Hand end.
Taking Lagrange's linear or quadratic interpolation function derive the appropriate element matrix equation.
Take 3 linear or 2 quadratic elements to get global matrix equation.
Find **Temperature Distribution** and **Heat Input Rate.**
Use the data given below :
K = 100, h = 50, D = 2 cm, Length L = 6 cms, θ_0 = 600, Compare your answers with exact.

2. (a) Say '**True** or '**False**' and justify your answer in brief. Rectify the statement if **False**. 10
(i) FEM is a method to solve a differential equation.
(ii) Shape function and Interpolation function are synonimous words.
(iii) Natural Boundary conditions are the values of primary variable.
(iv) In Galerkin method the weight function has a value 'o' at a point Essential Boundary condition is defined.
(v) Essential Boundary conditions are also called as Geometric Boundary conditions.

(b) Solve by Finite difference method, the following differential equation : 10

$$\frac{d^2u}{dx^2} - 25u + 10 = 0$$

B.C.S. u(0) = 0, u(2) = 4
Take 4 sub intervals.
Compare your answer at x = 0·5, 1, 1·5 with exact.

3. (a) Compare for advantages, the weak form methods over are non weak form methods. 4
(b) Explain : Degree of freedom, Dirichlet Boundary conditions. 3
(c) Solve the differential equation by using any **two** methods 13

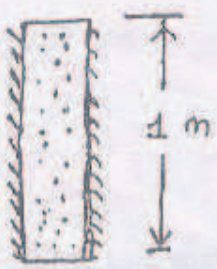
$$\frac{d^2u}{dx^2} - 16u - 10x^2 = 0$$

B.C.S. $u(0) = 0, \quad \frac{du}{dx}(1) = 1.$

(i) Collocation method
(ii) Subdomain method.
(iii) Galerking method.
(iv) Subdomain method.
(iv) Ritz method mapped over entire domain.

Compare your answers with exact at x = 0·3, 0·7, 1

4. Analyse completely any two : 20
(a) One dimensional fluid flow in porous medium.



Take 4 elements.

EME : $\left(\frac{AK}{L}\right)^e \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} p_1^e \\ p_2^e \end{Bmatrix} = \begin{Bmatrix} q_1^e \\ q_2^e \end{Bmatrix}$

K = permeability, cm/s
= 1·25 cm/s.

Fluid held at the top is 25 cm and that at the bottom is 2·5 cm. Area of cross-section is 5 cm^2.
Determine : (i) Fluid head distribution.
(ii) Velocity at the upper part.
(iii) Volumetric flow rate in upper part.

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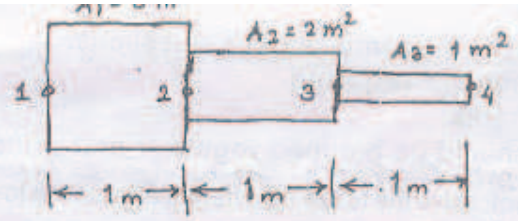


$$\left(\frac{AK}{L}\right)^e \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

K = permeability, m/s

q = discharge, m^3/s

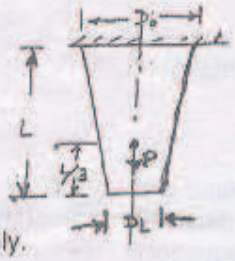
p = pressure head, m

$$\bar{v} = -K \times \frac{\Delta p}{L}$$

Data : $p_1 = 10$
$p_4 = 1$
K = 1 for all elements (smooth pipe)

Find : (a) Potential at the junctions,
(b) Velocities in each section of the pipe and
(c) Volumetric flow rate.

(c)

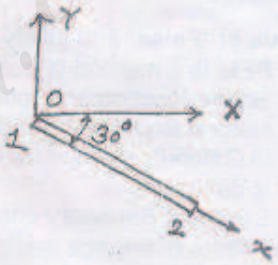


Data : E = 200 GPa
ρ = 60 kN/m³
P = 20 kN
L = 1·2 m
D_0 = 10 cm
D_L = 5 cm.
Square cross section

Analyse completely.

5. (a) Derive the oriented (w.r.t. GCS) stiffness matrix for an element of a plane truss. 6
(b) For the element shown below, global displacements have been determined to be– 4

U_1 = 10 mm, V_1 = 0·0
U_2 = 5 mm V_2 = 15 mm
E = 100 GPa, A = 200 mm², L = 100 cm
Determine : Elemental (local) displacements at each end



(c) Analysis the truss completely. 10

Data : E = 120 GPa for all elements
L = 3 m

A = 6 cm² for all elements
P_1 = 4 kN
P_2 = 6 kN

6. (a) Explain in Brief : (i) Effect of node numbering in FEA, (ii) h and p methods. 8
(b) Compare Newton cotes Method with Guass Quadrature Method. 4
(c) Find, using both the methods, M_{23} and K_{13}. 8

where $$M_{ij} = \int_0^{he} x^2 \phi_1 \phi_j \, dx$$

$$K_{ij} = \int_0^{he} x \frac{d\phi_i}{dx} \frac{d\phi_j}{dx} \, dx$$

where $$\phi_1 = \left(1 - \frac{x}{he}\right)\left(1 - \frac{2x}{he}\right)$$

$$\phi_2 = \frac{4x}{he}\left(1 - \frac{x}{he}\right)$$

$$\phi_3 = -\frac{x}{he}\left(1 - \frac{2x}{he}\right)$$

7. Attempt any **two** :– 20

(a) Beam Analysis.
(b) Error Analysis.
(c) Isoparametric Analysis.
(d) Software package used for FEA.
(e) Plane stress problems using CST element.