

(4 Hours)

[ Total Marks : 100

N.B. (1) Attempt any five questions.

(2) Assume any unspecified data if required.

1. (a) According to Newton's law of cooling, the rate at which the temperature of a body changes is proportional to the difference between the instantaneous temperature of the body and the temperature of the surrounding medium. If a body whose temperature is initially  $100^\circ\text{C}$  is allowed to cool in air which remains at the constant temperature  $20^\circ\text{C}$  and if it is observed that in 10 min. the body has cooled to  $60^\circ\text{C}$ , find the temperature of the body as a function of time. 12
- (b) Evaluate each complex function at the indicated value of  $s$  and determine its magnitude and phase. 8

$$(i) G(s) = \frac{(s+2)}{(s^2+s+1)} \text{ when } s = j2$$

$$(ii) F(s) = \frac{1}{s^2+3s+2} \text{ when } s = j2.$$

2. (a) Governing differential equation is given as : 8

$$\ddot{y} + 4\dot{y} = f(t) \quad \text{with } y(0) = \dot{y}(0) = 0$$

$$\text{where } f(t) = 0, t < 2$$

$$= t, t \geq 2$$

Solve using L.T. method.

State the theorems clearly.

- (b) The governing equation for a first order dynamic system is given as  $\dot{x} + 2x = \delta(t-1)$ . Assuming that the system is subjected to zero initial conditions, determine the response  $x(t)$ . Roughly sketch the graph of the response curve. 6
- (c) Repeat the problem Q. 2(b) for  $\dot{x} + x = u(t-1)$  subjected to zero initial conditions. 6
3. (a) Define system dynamics. 2
- (b) Describe the significance of system modelling in engineering design. Explain the steps involved in the modelling of a physical system. 6
- (c) Solve the differential equation 12

$$\ddot{y} + 4\dot{y} + 13y = \frac{1}{3} e^{-2t} \sin 3t$$

$$\text{for which } y(0) = 1 \text{ and } \dot{y}(0) = -2.$$

4. (a) Determine the inverse Laplace transform of 8

$$F(s) = \frac{2}{(s+3)(s^2+2s+5)}$$

Using (i) Partial Fraction Method

(ii) Convolution Method.

- (b) A mechanical system experiencing translational and rotational motion is given by 5

$$J\ddot{\theta} + B\dot{\theta} + K\theta + Rk(R\theta - x) = Ru(t)$$

$$m\ddot{x} + b\dot{x} - k(R\theta - x) = 0$$

Where  $J, K, B, R, m, k,$  and  $b$  denote parameters and are regarded as constants.  $x$  and  $\theta$  represent displacement and angular displacement, respectively and  $u(t)$  is an applied force. Express the equations in second order matrix form.

- (c) Governing differential equation is given as 7

$$2\ddot{x} + 3\dot{x} + x = f(t)$$

Obtain a state space form.

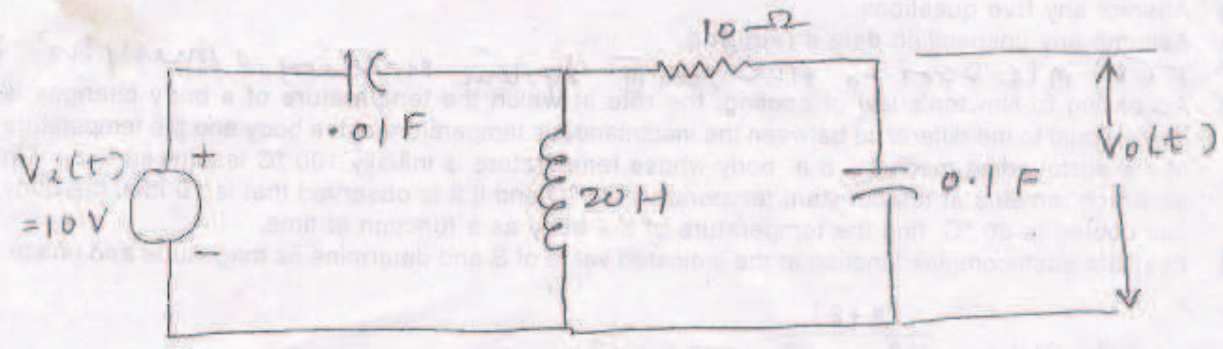
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5. (a) Find the state-space relation for the electric network shown below. Output is  $V_o(t)$

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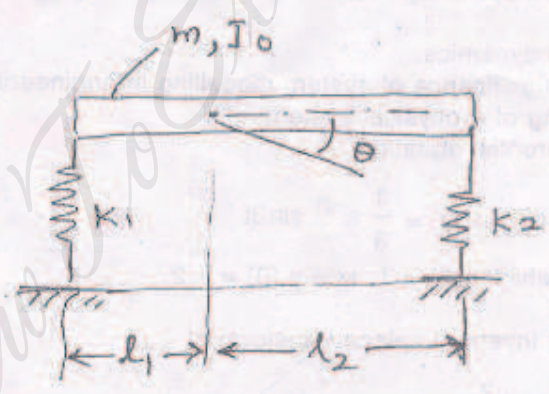
(b) Relate various parameters of a mechanical system to analogous electrical system using voltage force analogy. Name the law governing each system.

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- (c) Define :
- (i) State of dynamic system
  - (ii) State variable
  - (iii) State space
  - (iii) State equation
  - (iv) Output equation.

6. (a) A machine having mass  $m = 1500$  kg and a mass moment of inertia of  $J_0 = 400$  kg  $m^2$  is supported on elastic supports as shown in figure. If the stiffness of supports are  $K_1 = 2800$  N  $mm^{-1}$  and  $k_2 = 2100$  N  $mm^{-1}$ , and the supports are located at  $l_1 = 4$  m and  $l_2 = 5$  m, determine the natural frequencies and mode shapes of the machine tool.

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- (b) Explain the significance of :
- (i) Elastically conjugate points
  - (ii) Dynamically conjugate points
  - (iii) Doubly conjugate points.

7. (a) Consider a dynamic system with input  $f(t)$  and output  $x_1$ , whose state variable equations are

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$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = [-3x_2 - 2x_1 + f(t)]$$

Directly from these equations, determine input-output equation.

(b) Determine the transfer function for the single degree of freedom mechanical system shown in figure below using its state-space form.

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