

Con. 235-09.

TT-1020

(3 Hours)

[Total Marks : 100

- N.B. :** (1) Question No. 1 is **compulsory**.
 (2) Attempt any **four** questions from question Nos. 2 to question No. 7.
 (3) **Figures** to the **right** indicate **full** marks.

1. Solve : 20

- (a) $\frac{dy}{dx} = \cos(x+y)$
 (b) $(x - 2e^y) dy + (y + x \sin x) dx = 0$
 (c) $x(1-x^2)\frac{dy}{dx} + (2x^2 - 1)y = x^3$.
 (d) $(x^2 + y^2) dx + 2xy \cdot dy = 0$.

2. (a) Find the eigen values and eigen vector for— 10

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

(b) Find the inverse of the matrix by applying elementary row transformations 10

$$A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

3. (a) Find the n^{th} derivative by method of fraction : 10

$$y = \frac{x}{(x-1)(x-2)(x-3)}$$

(b) Find n^{th} derivative of 10

$$y = \frac{x}{x^2 + a^2}$$

4. (a) Find the curve in which length of the radius of curvature at any point is equal to two times the length of the normal at that point. 8

(b) Solve— 6

$$\frac{dy}{dx} - y \tan x = y^4 \sec x.$$

(c) From the differential equation from $x = a \sin (wt + c)$ where a and c are arbitrary constants. 6

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5. (a) Discuss the consistency of—

10

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

(b) Find the inverse of the following matrix by finding its adjoint :

10

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

6. (a) Find the order and degree of the following :

6

$$(1) y = x \frac{dy}{dx} + \frac{5}{\frac{dy}{dx}}$$

$$(2) y = x \frac{dy}{dx} + 5 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

(b) Reduce the matrix to Echelon and find its rank—

8

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

(c) Solve—

6

$$(x + y) \frac{dy}{dx} + y = 0.$$

7. (a) Solve—

6

$$(y^4 + 2y) dx (xy^3 + 2y^4 - 4x) dy = 0.$$

(b) If the temperature of air is 30 °C and substance cools from 100°C to 70°C in 15 minutes. Find when the temperature will be 40°C.

8

(c) If $z = x^y + y^x$ then show that—

6

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}.$$