

MASTER

- N.B. : (1) Question No. 1 is compulsory.
 (2) Attempt any four questions from remaining six questions.
 (3) Figures to the right indicate full marks.

S. K. SINGH *18/6/08*
 1. (a) State and prove Parseval's identity over $(c, c + 2l)$. 5

(b) Find orthogonal trajectories of the family of curves $e^{-x} \cos y + xy = \text{const.}$ 5

(c) Find $L^{-1} \left\{ \frac{2}{(s+1)^2 (s^2+4)} \right\}$. 5

(d) S.T. every square matrix can be uniquely expressed as a sum of Hermitian and Skew-Hermitian matrix. 5

2. (a) Solve using Laplace transform - 6

$$\frac{dy}{dt} + 2y + \int_0^t y dt = \sin t. \text{ given } y(0) = 1.$$

(b) Find analytic function $f(z)$ whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$. 6

(c) (i) If 'A' is a nonsingular square matrix of order 'n' then S.T. $\text{adj} \cdot (\text{adj} \cdot A) = |A|^{n-2} \cdot A$. 4

(ii) Verify $A(\text{adj} \cdot A) = |A| I$ 4

Where $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$

3. (a) Use Laplace transform to evaluate - 6

$$\int_0^\infty \int_0^t e^{-t} \frac{\sin u}{u} du dt$$

(b) S.T. every bilinear transformation is the resultant of three basic transformations. 6

(c) Find Fourier series for - 8

$$f(x) = \left(\frac{\pi-x}{2} \right)^2 \text{ over } (0, 2\pi).$$

Hence S.T. (i) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots$

(ii) $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots$

4. (a) Find A^{-1} by using elementary transformations - 6

Where $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$

App. marks - 121 - 187 2/10

(b) Find Fourier series for - 6
 $f(x) = x \cos x ; -\pi < x < \pi.$

(c) S.T. the relation $W = \frac{iz + 2}{4z + i}$ transforms the real axis in z-plane into a circle in w-plane. 8
Find its centre and radius. Also find the point in z-plane which is mapped on the centre of the circle in w-plane.

5. (a) For what values of λ and μ the system 6
 $x + y + z = 6$
 $x + 2y + 3z = 10$
 $x + 2y + \lambda z = \mu$
has (i) no solution (ii) unique solution (iii) more than one solutions.
Also find parametric solution.

(b) Find half range Cosine series for 6
 $f(x) = \begin{cases} 1 & 0 < x < 1 \\ x & 1 < x < 2 \end{cases}$

(c) Find (i) $L^{-1} \left\{ \frac{e^{-\pi s}}{s^2 - 2s - 2} \right\}$ 4

(ii) $L^{-1} \left\{ \frac{1}{(s^2 - q^2)^2} \right\}$ 4

6 (a) If $f(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & 1 < t < 2 \end{cases}$ 6
and $f(t) = f(t + 2)$ then

S. T. $L \{f(t)\} = \frac{1}{s(1 + e^{-s})}$

(b) Define orthogonal and orthonormal set of functions. S.T. $\{ \sin n x \}_{n = 1, 2, 3, \dots}$ is 6
orthogonal set of functions over $[0, \pi]$. Hence construct orthonormal set of functions.

(c) Find Fourier series for 8
 $f(x) = 2x - x^2 ; 0 \leq x \leq 2.$

7. (a) Find complex form of Fourier series for - 6
 $f(x) = e^{ax} ; (-L, L).$

(b) Find (i) $L \left\{ \frac{\sqrt{1 + \sin 4t}}{e^{2t}} \right\}$ 3

(ii) $L \{ (t \sin 2t)^2 \}.$ 3

(c) State convolution theorem and use it to find $L^{-1} \left\{ \frac{s}{(s^2 + 4)^2} \right\}.$ 8

OR

(c) Define cross ratio. Find Bilinear transformation which transforms point $z = i, 1, -1$ into 8
 $w = 1, 0, \infty$ respectively.