S.E. (MECH.) Sem IV (Rev) . 19/6/08

Lib.

Computational Mathematics

Con. 3539-08. **CO-9703**

(REVISED COURSE)

(3 Hours) [Total Marks : 100

N.B. (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assume **suitable** data, wherever **required** and justify the **same**.
(4) **Figures** to the **right** indicate **full** marks.

S.E. (M) Sem IV Rev Comp. Maths – 19/6/08

1. (a) Find the function f(x) whose first difference is $9x^2 + 11x + 5$. 20

(b) Show that the set of functions $P_0(x) = 1$, $P_1(x) = x$, $P_2(x)\ \frac{1}{2}(3x^2 - 1)$ are orthogonal over $(-1, 1)$.

(c) Using Taylor series find the solution of $x\frac{dy}{dx} = x - y$, $y(2) = 2$ at $x = 2{\cdot}1$ correct to four decimal places.

(d) Using the method of separation of variables solve :

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0 \quad \text{with } u(x, 0) = 4e^{-x}.$$

2. (a) Obtain all possible solution of Laplace equation in polar co-ordinates, 6

$$r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$$

(b) Find the number of students from the following data who secured marks not more than 45, using interpolation : 6

Marks	30—40	40—50	50—60	60—70	70-80
No. of students	35	48	70	40	22

(c) Using R-K method of fourth order, find y(0·2) correct to four decimal places if 8

$$\frac{dy}{dx} = xy + y^2,\ y(0) = 1 \quad \text{with two steps.}$$

3. (a) Find half range cosine series for the function $f(x) = (x - 1)^2$ in $(0, 1)$. Hence deuce that 6

$$\pi^2 = 6\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + ...\right)$$

(b) When a train is moving at 30 m/sec, steam is shut off and brakes are applied. The speed of the train per second after 't' seconds is given below : 6

Time (t)	0	5	10	15	20	25	30	35	40
Speed (V)	30	24	19·5	16	13·6	11·7	10·0	8·5	7·0

Using Simpson's rule determine the distance moved by train in 40 seconds.

(c) A tightly stretched string with fixed end points x = 0 and x = L is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its point a velocity 8

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = 3\,(Lx - x^2) \quad \text{find } y\,(x, t).$$

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4. (a) Apply Lagrange's interpolation formula to fit a polynomial to the data given below : 6

x	0	1	3	4
y	− 12	0	6	12

Also find the value of y at x = 2.

(b) Evaluate $\int_0^5 \frac{dx}{4x+5}$ by Trapezoidal rule using 11 co-ordinates. 6

(c) Find Fourier series of the function 8

$$f(x) = \pi + x, \quad -\pi < x < 0$$
$$= 0 \;, \quad 0 < x < \pi$$

Hence deduce that—

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$$

5. (a) Using Gauss-seidal iteration method solve the system of equations : 6

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

Use three iterations.

(b) Find half range sine series for the function $f(x) = x^2 - 2,\ 0 < x < 2.$ 6

Hence deduce that—

$$\pi^3 = 32\left(1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + ...\right)$$

(c) With usual notations prove that— 8

(i) $(E+1)\delta = 2(E-1)\mu$

(ii) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \delta^2/4}$.

6. (a) Find a real positive root of the equation $x^3 - 7x + 5 = 0$ by using bisection method correct to 3 places of decimal. 6

(b) Express the function : 6

$$f(x) = \begin{cases} 1 & \text{for } 0 \le x \le \pi \\ 0 & \text{for } x > \pi \end{cases}$$

as a Fourier Sine integral. Hence evaluate

$$\int_0^\infty \frac{1-\cos \pi\lambda}{\lambda} \cdot \sin(x\lambda)\, d\lambda .$$

(c) A bar AB of length 10 cm has its ends A and B kept at 30° and 100° temperatures respectively until steady state condition is reached. Then the temperature at A is lowered to 20° and that at B to 40° and these temperatures are maintained. Find subsequent temperature distribution in the bar. 8

7. (a) Find the real root of 3x − cosx − 1 = 0 by Newton-Raphson method correct to four decimal places. 6

(b) Obtain complex form of Fourier series of $f(x) = e^{ax}$ in $(0, 2\pi)$. 6

(c) A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered infinite in the length without introducing an appreciable error. If the temperatures along one short edge y = 0 is given by $100 \sin\left(\frac{\pi x}{8}\right)$; while the two long edges x = 0 and x = 8 as well as the other short edge are kept at 0°C, find the steady state temperature function u(x, y). 8