

- N.B. :** (1) Question No. 1 is compulsory.
 (2) Attempt any four questions out of remaining six.
 (3) Non-programmable calculator is allowed.
 (4) Write the sub questions of the main question collectively,

1. (a) Find Z-transform of $\cos\left(\frac{\pi k}{3} + \alpha\right)$, $k \geq 0$ 5

(b) State and prove change of scale property. 5

(c) Find A^{-1} using adjoint method where 5

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

(d) Find the complex form of the Fourier series for $f(x) = e^{-x}$ in $(-1, 1)$. 5

2. (a) Evaluate $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$. 6

(b) Obtain the fourier series for $f(x) = x \cos x$ in $(-\pi, \pi)$. 6

(c) If A, B, C are non-singular matrices of order $n \times n$ then prove that. 8

$$\begin{bmatrix} A & O \\ B & C \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} & O \\ -C^{-1}BA^{-1} & C^{-1} \end{bmatrix}$$

Hence find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 3 & 0 & 4 & 0 \\ 0 & 4 & 0 & 2 \end{bmatrix}$

3. (a) Using convolution Th^m find the Laplace inverse transform of

6

$$\frac{5}{(s^2 + a^2)(s^2 + b^2)}$$

(b) Show that every square matrix can be uniquely expressed as the sum of symmetric and skew-symmetric matrices.

6

(c) Find Fourier cosine integral for

8

$$\begin{aligned} f(x) &= 1 - x^2, & 0 \leq x \leq 1 \\ &= 0, & x > 1 \end{aligned}$$

Hence evaluate $\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$.

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4. (a) Using L.T. solve, $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1$, $y(0) = 0$, $y'(0) = 1$. 6

(b) Find the non-singular matrices P and Q such that PAQ is in normal form. Also find its rank. Where 6

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 1 & 0 & 6 \end{bmatrix}$$

(c) Express as a Fourier series, 8
 $f(x) = \pi x$, $0 \leq x \leq 1$
 $= \pi(2-x)$, $1 \leq x \leq 2$.

5. (a) Find L {f(t)} where $f(t) = \sin 2t$, $0 \leq t \leq \frac{\pi}{2}$, $f(t) = 0$, $\frac{\pi}{2} \leq t \leq \pi$ and $f(t) = f(t + \pi)$. 6

(b) Obtain half-range cosine series for $f(x) = (x-1)^2$ in $0 < x < 1$. 6

(c) Find the inverse Z-transform of $f(z) = \frac{1}{(z-3)(z-2)}$. 8

If region of convergence is (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$.

6. (a) Show that the set of functions 6

$$\sin\left(\frac{\pi x}{2l}\right), \sin\left(\frac{3\pi x}{2l}\right), \sin\left(\frac{5\pi x}{2l}\right), \dots$$

is orthogonal over $(0, l)$.

(b) Find Z $\{2^k \sin(3k+2)\}$, $k > 0$. 6

(c) Find the inverse L.T. of - 8

(i) $e^{-s} \left\{ \frac{1-\sqrt{s}}{s^2} \right\}^2$ (ii) $\frac{s+29}{(s+4)(s^2+9)}$

(a) Test the consistency of the following equations and solve them if they are consistent. **6**

$$\begin{aligned}x + y + z &= 6, & x - y + 2z &= 5, \\3x + y + z &= 8, & 2x - 2y + 3z &= 7.\end{aligned}$$

(b) Find the Fourier series for **6**

$$\begin{aligned}f(x) &= 0, & -\pi < x < 0 \\&= \sin x, & 0 < x < \pi\end{aligned}$$

Hence deduce that,

$$\frac{1}{2} = \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots$$

(c) Find the L. T. of **8**

(i) $\frac{\sin t \sinh t}{t}$

(ii) $f(t) = \cos t, \quad 0 < t < \frac{\pi}{2}$
 $= \sin t, \quad t > \frac{\pi}{2}.$

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