F.E. I (old) (All Br) 7/12/07

2hf-Nov-Nk-07-52

App. Maths. I Dec' 07 MASTER

Con. 4950-07. **CD-6237**

(OLD COURSE)

(3 Hours) [Total Marks : 100

N.B. (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicate marks.

1. (a) Solve the equation 17 coshx + 18 sinhx = 1, for real values of x. 5

(b) If $\bar{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + at\tan\alpha\,\hat{k}$ then prove that— 5

$$\left|\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2}\right| = a^2 \sec\alpha .$$

(c) Find the n^{th} derivative of $\frac{x^2}{(x+2)(2x+3)}$. 5

(d) If $u = \left(1-2xy+y^2\right)^{-\frac{1}{2}}$ then 5

Prove that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2 u^3$.

2. (a) Find the cube roots of unity. If w is a complex cube root of unity then prove that— 8

(i) $1 + w + w^2 = 0$

(ii) $\frac{1}{1+2w} + \frac{1}{2+w} - \frac{1}{1+w} = 0.$

(b) Using De'Moivre's theorem solve the equation 6

$x^7 + x^4 + ix^3 + i = 0.$

(c) If cos (x + iy) = cosα + i sinα then P.T. 6

$\sin\alpha = \pm\sin^2 x = \pm\sin h^2 y.$

3. (a) Find the equations of the osculating plane and rectifying plane to the curve 8

$x = 2\log t, \quad y = 4t, \quad z = 2t^2 + 1$ at t = 1

(b) If $i^{i^{i^{\cdots\infty}}} = A + iB$, considering the principal values proves that— 6

$\tan\left(\frac{\pi A}{2}\right) = \frac{B}{A}$ and $A^2 + B^2 = e^{-\pi B}$.

(c) Show that— 6

$$\tan\left[i\log\left(\frac{a-bi}{a+bi}\right)\right] = \frac{2ab}{a^2-b^2}.$$

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ZHE-NOV-NK-07-53

Applied Maths I, Dec 07

4. (a) Find the curvature, torsion, radius of curvature and radius of torsion for the curve 8

$\bar{r} = 3t\,\hat{i} + 3t^2\,\hat{j} + 2t^3\,\hat{k}$.

(b) Apply Taylor's theorem to expand 6

$x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x - 1)$

(c) State Rolle's Theorem and verify the same for 6

$f(x) = x^2 (1 - x)^2$ in $0 \le x \le 1$.

5. (a) State and prove Euler's Theorem for function of two variables and verify the same for 8

$$u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y} .$$

(b) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{2x + 3y}\right)$, prove that 6

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u .$$

(c) Find the stationary values of 6

$x^3 + y^3 - 3\,axy$; $a > 0$

6. (a) P.T. $e^{\cos^{-1}x} = e^{\frac{\pi}{2}}\left[1 - x + \dfrac{x^2}{2} - \dfrac{x^3}{3} + ...\right]$ 8

(b) Expand $y = \cos^7 x$ in cosines of multiples of x and then find y_n. 6

(c) Find the values of a, b if 6

$$\lim_{x \to 0} \frac{a \sinh x + b \sin x}{x^3} = \frac{5}{3} .$$

7. (a) If $z = f(x, y)$; $x = e^u \cos v$, $y = e^u \sin v$ then 8

Prove that—

(i) $x \dfrac{\partial z}{\partial v} + y \dfrac{\partial z}{\partial u} = e^{2u} \dfrac{\partial z}{\partial y}$

(ii) $\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\dfrac{\partial z}{\partial u}\right)^2 + \left(\dfrac{\partial z}{\partial v}\right)^2\right]$.

(b) If $y = -18 \cos(\log x) + 17 \sin(\log x)$ 6

Prove that $x^2 y_{n+2} + (2n + 1)\,xy_{n+1} + (n^2 + 1)\,y_n = 0$.

(c) In calculating the volume of a cone errors of 2% and 1% are found in measuring height and base radius respectively. Find the percentage error in calculating volume. 6