(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicates **full** marks.
(4) Answers should be **grouped** and written **one** below the **other**.

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1. (a) If $y = e^{ax}\cos(bx + c)$ proved that $y_n = \left(a^2 + b^2\right)^{n/2} e^{ax} \cos\left(bx + c + n\tan^{-1}(b/a)\right)$ and hence find $n^{th}$ derivative of $e^{5x}\cos x \cos 3x$. 5

(b) Prove that :– 5

$$(\bar{b} \times \bar{c}) \times (\bar{a} \times \bar{d}) + (\bar{c} \times \bar{a}) \times (\bar{b} \times \bar{d}) + (\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = -2[\bar{a}\ \bar{b}\ \bar{c}]\ \bar{d}$$

(c) If $u = A\, e^{-gx}\sin(nt - gx)$ satisfies the equation $\dfrac{\partial u}{\partial t} = \mu \dfrac{\partial^2 u}{\partial x^2}$, 5

Prove that $g = \sqrt{\dfrac{n}{2\mu}}$

(d) Prove that the statements $Re z > 0$ and $|z - 1| < |z + 1|$ are equilvalent where $z = x + iy$. 5

2. (a) Show that the roots of $z^5 = 1$ can be written as $1, u, u^2, u^3, u^4$. Hence prove that $(1 - u)(1 - u^2)(1 - u^3)(1 - u^4) = 5$ 6

(b) Prove that –

(i) $\cosh^{-1}\left(\sqrt{1+x^2}\right) = \sinh^{-1} x$ 3

(ii) $\text{Log}\left[\dfrac{\sin(x + iy)}{\sin(x - iy)}\right] = 2i\tan^{-1}[\cot x \tanh y]$ 4

(c) If $u = \operatorname{cosec}^{-1}\left[\sqrt{\dfrac{\sqrt{x} + \sqrt{y}}{x^{1/3} + y^{1/3}}}\right]$ prove that 7

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{12}\tan u\left[\frac{\tan^2 u}{12} + \frac{13}{12}\right]$$

3. (a) Find all stationary values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ 6

(b) If $y = \sin rx + \cos rx$, prove that 7

$$y_n = r^n\left[1 + (-1)^n \sin 2rx\right]^{1/2} \quad \text{find } y_8(\pi) \text{ where } r = \frac{1}{4}$$

(c) For the curve $x = t\cos t$, $y = t\sin t$, $z = at$, Find radius of curvature and torsion at $t = 0$. 7

4. (a) Expand $\dfrac{x}{e^x - 1}$ upto $x^4$ and hence show that – 6

$$\frac{x}{2}\frac{\left(e^x + 1\right)}{\left(e^x - 1\right)} = 1 + \frac{1}{12}x^2 - \frac{1}{720}x^4 + \ldots\ldots$$

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(b) If $\frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1$, where u is a homogeneous function of degree n in x, y, z prove that 7

$$u_x^2+u_y^2+u_z^2=2nu$$

(c) If $x=\sqrt{vw}$, $y=\sqrt{wu}$, $z=\sqrt{uv}$ prove that 7

$$x\frac{\partial\phi}{\partial x}+y\frac{\partial\phi}{\partial y}+z\frac{\partial\phi}{\partial z}=u\frac{\partial\phi}{\partial u}+v\frac{\partial\phi}{\partial v}+w\frac{\partial\phi}{\partial w},$$ where $\phi$ is a funciton of x, y, z

5. (a) If $x_n=\cos\left(\frac{\pi}{2^n}\right)+i\sin\left(\frac{\pi}{2^n}\right)$ Then show that 6

   (i) $x_1\, x_2\, x_3 \ldots\ldots\ldots x_\infty = -1$

   (ii) $x_0\, x_1\, x_2 \ldots\ldots\ldots x_\infty = 1$

(b) If $z_1$ and $z_2$ are two complex numbers such that $|z_1+z_2|=|z_1-z_2|$. Then prove that $\arg z_1-\arg z_2=\frac{\pi}{2}$ 7

(c) If F, $\phi$, $\psi$ are continuous functions in [a, b] and derivable in (a, b). Then show that there is a value 'c' lying between 'a' and 'b' 7

such that $\begin{vmatrix} f(a) & f(b) & f'(c) \\ \phi(a) & \phi(b) & \phi'(c) \\ \psi(a) & \psi(b) & \psi'(c) \end{vmatrix}=0$

6. (a) Prove that $\operatorname{Log}\left[\frac{(a-b)+i(a+b)}{(a+b)+i(a-b)}\right]=i\left[2n\pi+\tan^{-1}\left(\frac{2ab}{a^2-b^2}\right)\right]$ 6

(b) Find the constants a, b, c so that $\lim\limits_{x\to 0}\frac{x(a+b\cos x)-c\sin x}{x^5}=1$ 7

(c) If $y=\frac{1}{1+x+x^2}$ prove that $y_n=\frac{2(-1)^n}{\sqrt{3}}\,\frac{n!}{r^{n+1}}\sin(n+1)\theta$ where 7

$\theta=\cot^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$ and $r=\sqrt{1+x+x^2}$

7. (a) Find the approximate value of $\left[(0\cdot 98)^2+(2\cdot 01)^2+(1\cdot 94)^2\right]^{\frac{1}{2}}$ 6

(b) If $\phi\left(\frac{z}{x^3},\frac{y}{x}\right)=0$ Then prove that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=3z$ 7

(c) A tangent to a curve makes a constant angle $\phi$ with a Fixed line. Show that $\frac{k}{\tau}$ = constant. Where k is curvature and $\tau$ is torsion of space vector. 7