(REVISED COURSE)

20/5/08

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **Compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.

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1. (a) If $u = \log\left(\tan\left[\frac{\pi}{4} + \frac{\theta}{2}\right]\right)$ then P.T. **20**

(i) $\cosh u = \operatorname{Sec}\theta$
(ii) $\sinh u = \tan\theta$.

(b) Find the complex number 'z' if—

$$\arg(z + 1) = \frac{\pi}{6} \text{ and } \arg(z - 1)\ \frac{2\pi}{3}.$$

(c) If $u = (1 - 2xy + y^2)^{-1/2}$, P.T. $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$.

(d) If $\bar{u} = a\cos t\, i + a\sin t\, j + at\tan\alpha\, k$

then S.T. $\left[\frac{d\bar{u}}{dt}\ \ \frac{d^2\bar{u}}{dt^2}\ \ \frac{d^3\bar{u}}{dt^3}\right] = a^3\tan\alpha$.

2. (a) If $y = e^{m\sin^{-1}x}$ then P.T. **6**

$$(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - (n^2 + m^2)\, y_n = 0.$$

(b) Find the maximum and minimum values of— **6**

$$x^3 + 3xy^2 - 3x^2 - 3y^2 + 4.$$

(c) If $z = f(x, y)$, $x = e^u\cos v$, $y = e^u\sin v$ **8**

P.T. (i) $x\frac{\partial z}{\partial v} + y\frac{\partial z}{\partial u} = e^{2u}\frac{\partial z}{\partial y}$

(ii) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right]$

3. (a) Prove that $\nabla(f(r)) = f'(r)\frac{\bar{r}}{r}$ and hence find f if $\nabla f = 2r^4\bar{r}$. **6**

(b) Find the values of a, b, c so that— **6**

$$\lim_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2.$$

(c) If $\cos\alpha + \cos\beta + \cos y = 0$ and $\sin\alpha + \sin\beta + \sin y = 0$ then P.T. :— **8**

(i) $\sin^2\alpha + \sin^2\beta + \sin^2 y = \cos^2\alpha + \cos^2\beta + \cos^2 y = \frac{3}{2}$.

(ii) $\cos(2\alpha) + \cos(2\beta) + \cos(2y) = 0$

(iii) $\cos(\alpha + \beta) + \cos(\beta + y) + \cos(y + \alpha) = 0$

(iv) $\sin(\alpha + \beta)\sin(\beta + y) + \sin(y + \alpha) = 0$.

[TURN OVER

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4. (a) If $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, P.T. **6**

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

(b) If $x + iy = c\cot(u + iv)$, Show that— **6**

$$\frac{x}{\sin(2u)} = \frac{-y}{\sinh(2v)} = \frac{c}{\cosh(2v) - \cos(2u)}$$

(c) If $\bar{a}$ is a constant vector and $\bar{r} = xi + yj + zk$ **8**

P.T. (i) $\text{div}\,(\bar{a}\times\bar{r}) = 0$

(ii) $\text{div}\,(\bar{a}\cdot\bar{r})\bar{a} = a^2$

(iii) $(\bar{a}\times\bar{r}\times\bar{a}) = 2a^2$

(iv) $\text{curl}\,(\bar{a}\times\bar{r}) = 2\bar{a}.$

5. (a) P.T. $\tan^y x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots$ **6**

(b) P.T. $\frac{b-a}{b} < \log\left(\frac{b}{a}\right) < \frac{b-a}{a}$ for $0 < a < b$ **6**

Hence deduce that $\frac{1}{4} < \log\frac{4}{3} < \frac{1}{3}.$

(c) Separate into real and imaginary parts $\tan^{-1}(\cos\theta + \sin\theta)$ **8**

6. (a) Test the convergence of— **6**

$$\frac{x}{1\cdot 2} + \frac{x^2}{3\cdot 4} + \frac{x^3}{5\cdot 6} + \frac{x^4}{7\cdot 8} + \ldots\ldots \quad (x > 0 \text{ and } x \neq 1)$$

(b) If $u = f(y/x) + \sqrt{x^2 y^2}$ **6**

then P.T. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sqrt{x^2 + y^2}.$

(c) If $y = 2^x \cos^9 x$ then find y_n. **8**

7. (a) Find all roots of $(x + 1)^7 = (x - 1)^7$. **6**

(b) If $u = \text{cosec}^{-1}\left(\sqrt{\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}\right)$ then P.T. **6**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left[\frac{13}{12} + \frac{\tan^2 u}{12}\right]$$

(c) If $z = x\log(x + r) - r$ where $r^2 = x^2 + y^2$ **8**

P.T. (i) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{x + r}$

(ii) $\frac{\partial^2 z}{\partial x^3} = -\left(\frac{x}{r^3}\right)$