lib F.E. (All br.) II (Rev) 8/12/07

App. Maths. II, Sg. 1, Dec 07

Con. 4837-07. CD-5482

(3 Hours) [Total Marks : 100

De MA-3052

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining **six**.

1. (a) Prove that $\left|\frac{3}{2}-x\right.\ \left|\frac{3}{2}+x\right.=\left(\frac{1}{4}-x^2\right)\pi\sec\pi x$ provided $-1<2x<1$. 5
   (b) Solve $(x^2+y^2+1)\,dx-2xy\,dy=0$. 5
   (c) Show that $\int_0^\infty \frac{\log(1+ax^2)}{x}\,dx=\pi\sqrt{a}\ \ (a>0)$. 5
   (d) Find the total length of the curve $x^{2/3}+y^{2/3}=a^{2/3}$. 5

2. (a) Prove that $\int_0^\pi \frac{\sin^{n-1}x}{(a+b\cos x)^n}\,dx=\frac{2^{n-1}}{(a^2-b^2)^{n/2}}\,\beta\left(\frac{n}{2},\frac{n}{2}\right)$ 6
   (b) Change the order of integration and evaluate 6
   $$\int_0^1\int_0^{\sqrt{1-y^2}}\frac{\cos^{-1}x}{\sqrt{1-x^2}\,\sqrt{1-x^2-y^2}}\,dx\,dy.$$
   (c) Solve $(D^3+1)y=e^{\frac{x}{2}}\sin\left[\frac{\sqrt{3}}{2}x\right]$. 8

3. (a) Find by double integration the area enclosed by the curve $9xy=4$ and the line $2x+y=2$. 6
   (b) Evaluate $\iint_R \frac{dx\,dy}{x^4+y^2}$ where R is the region $x\ge 1$ and $y\ge x^2$. 6
   (c) Solve $(D^2-D)y=e^x\sin x$ by method of undetermined coefficient. 8

4. (a) Find the volume bounded by $y^2=x$, $x^2=y$ and the planes $z=0$ and $x+y+z=1$. 6
   (b) Evaluate $\int_0^\infty x\,e^{-x^8}\,dx\ \int_0^\infty x^2\,e^{-x^4}\,dx$. 6
   (c) Solve $(D^2-1)y=2\left(1-e^{-2x}\right)^{\frac{-1}{2}}$ by variation of parameter. 8

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5. (a) Find the mass of the lamina bounded by the curve $ay^2 = x^3$ and the line $by = x$. If the density at a point varies as the distance of the point from the x axis. 6

(b) Verify the rule of D.U.I.S. for $\int_0^{\infty} e^{-at} \sin bt\,dt$. 6

(c) Solve $\left(\frac{d}{dx} + \frac{1}{x}\right)^2 y = \frac{1}{x^4}$. 8

6. (a) Evaluate $\iiint x^2yz\,dxdydz$ throughout the volume bounded by $x = 0$ $y = 0$ $z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. 6

(b) Find the length of the upper arc of one loop of lemniscate $r^2 = a^2\cos 2\theta$. 6

(c) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$. 8

7. (a) Change to polar co-ordinates and evaluate $\iint_R \frac{1}{\sqrt{xy}}\,dxdy$ where R is the region of Integration bounded by $x^2 + y^2 - x = 0$. 6

(b) Solve $\frac{dy}{dx} = x^3 y^3 - xy$. 6

(c) Prove that $\beta\left(n + \frac{1}{2}, n + \frac{1}{2}\right) = \frac{1}{2^{2n}} \frac{\left|\overline{n + \frac{1}{2}}\right.}{\left|\overline{n+1}\right.} \sqrt{\pi}$ 8

deduce that $2^n \left|\overline{n + \frac{1}{2}}\right. = 1.3.5....(2n-1)\sqrt{\pi}$.