F E (All Br.) Sem-II (Rev.)
Applied Maths-II, May '07 25/05/07 11 to 2

C/n-2688-07. ND-995

(REVISED COURSE)

( 3 Hours ) [ Total Marks : 100

MASTER

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining **six**.
(3) **Figures** to the **right** indicate marks.

1. (a) State Duplication formula for Gamma function and prove that— 20

$$1 \cdot 3 \cdot 5 \cdot 7 \ldots\ldots\ldots\ldots (2n-1) = \frac{2^n \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}}.$$

(b) Solve : $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$.

(c) Evaluate $\iiint_V (x^2+y^2+z^2)\ dx\ dy\ dz$ over the first octant of the sphere $x^2+y^2+z^2=a^2$.

(d) Sketch the region bounded by the curves $xy = 16$, $y = x$, $x = 8$ and $y = 0$. Express area of this region as a double integral in two ways.

2. Solve the following differential equations.

(a) $\left(xy^2 - e^{1/x^3}\right) dx - x^2y\ dy = 0$. 4

(b) $\frac{dr}{d\theta} = r \tan\theta - \frac{r^2}{\cos\theta}$. 4

(c) $(D^2 - 3D + 2)\ y = 2\,e^x \sin\left(\frac{x}{2}\right)$. 6

(d) $(D^3 - 7D - 6)\ y = (1 + x^2)\ e^{2x}$. 6

3. (a) Find the length of loop of the curve $3ay^2 = x(x-a)^2$. 6
(b) Use the rule of D.U.I.S. to prove that— 6

$$\int_0^\infty e^{-\left(x^2+\frac{a^2}{x^2}\right)} dx = \frac{\sqrt{\pi}}{2} e^{-2a}, a>0.$$

(c) Change the order of integration and Evaluate. 8

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\ dx\ dy.$$

4. (a) Evaluate $\iint_R \frac{dx\ dy}{\left(1+x^2+y^2\right)^2}$ over one loop of Leminiscate 6

$(x^2+y^2)^2 = x^2 - y^2$ by converting into polar co-ordinates.
(b) Find the volume bounded by paraboloid $x^2+y^2 = 4z$ and the cylinder $x^2+y^2 = 16$. 6
(c) Prove that— 8

(i) $\int_0^\infty \frac{e^{-x^3}}{\sqrt{x}}\,dx \times \int_0^\infty y^4 e^{-y^6}\,dy = \frac{\pi}{9}$.

(ii) $\int_0^\infty \frac{dx}{\left(e^x+e^{-x}\right)^n} = \frac{1}{4}\beta\left(\frac{n}{2},\frac{n}{2}\right)$.

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5. (a) Show that the line $\theta = \frac{\pi}{3}$ divides the length of astroid $x^{2/3} + y^{2/3} = a^{2/3}$ in first quadrant in the ratio 1:3. 6

(b) Solve by the method of undetermined multipliers. 6

$$(D^2 - 4D + 4)\ y = x^2 + e^x + \cos 2x.$$

(c) (i) Evaluate $\iint_R xy\ dx\ dy$ where R is bounded by circle $x^2 + y^2 = 2x$, parabola $y^2 = 2x$ and the line $y = x$. 5

(ii) Find the area of the region common to two circles $r = a\cos\theta$ and $r = a\sin\theta$ by double integration. 3

6. (a) Evaluate $\int_0^{\pi} \frac{dx}{a+b\cos x}$, $\begin{matrix} a>0 \\ b>0 \end{matrix}$ and applying rule of D.U.I.S. deduce that— 6

(i) $$\int_0^{\pi} \frac{dx}{(a+b\cos x)^2} = \frac{\pi a}{\left(a^2-b^2\right)^{3/2}}$$

(ii) $$\int_0^{\pi} \frac{\cos x\, dx}{(a+b\cos x)^2} = \frac{-\pi b}{\left(a^2-b^2\right)^{3/2}}.$$

(b) Find the mass of Lamina in the form of cardioide $r = a(1-\cos\theta)$ if the density at any point varies as it's distance from the pole. 6

(c) Solve : 8

$$x^2\frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{y}{x} = 4\log x.$$

7. (a) Prove that— 6

$$\int_0^1 \frac{\left(1-x^4\right)^{3/4}}{\left(1+x^4\right)^2}\,dx = \frac{1}{4}\,\frac{1}{2^{1/4}}\,\beta\left(\frac{7}{4},\frac{1}{4}\right)$$

(b) Evaluate — 6

$$\iiint_V \frac{dx\,dy\,dz}{(1+x+y+z)^3}$$

over the volume of tetrahedron bounded by planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

(c) Solve by the method of variation of parameters. 8

$$\frac{d^2y}{dx^2} + y = \frac{1}{1+\sin x}.$$