UG–446/UG–449 **BMS–12/ BMC–12**

B.Sc. DEGREE EXAMINATION – JULY 2008.

Mathematics/Mathematics with Computer Application

First Year

TRIGONOMETRY, ANALYTICAL GEOMETRY (3D) AND VECTOR CALCULUS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Prove that $\sinh 2x = \dfrac{2\tanh x}{1-\tanh^2 x}$.

2. Evaluate $\lim\limits_{x\to 0} \dfrac{\tan 2x - 2\tan x}{x^3}$.

3. Find the equation of the plane through the point (1, 2, 4) and parallel to the plane $2x + 6y - 8z + 9 = 0$.

4. Find the angle between the straight lines

$$\frac{x-1}{4} = \frac{y+1}{3} = \frac{z-1}{1} \text{ and } \frac{x-2}{3} = \frac{y+4}{1} = \frac{z-4}{5}.$$

5. Find the equation of the sphere having centre at (7, 4, –3) and radius 6.

6. Show that

$$\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$$

is irrotational and solenoidal.

7. If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve on the xy plane $y = 2x^2$ from (0, 0) to (1, 2).

8. Prove that $div\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$, if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. If $\tan \frac{x}{2} = \tanh \frac{y}{2}$ prove that

(a) $\sinh y = \tan x$ and

(b) $y = \log \tan \left(\frac{\pi}{4} + \frac{x}{2}\right)$.

10. Sum to n terms the series

$$\sin^2 \alpha + \sin^2 2\alpha + \sin^2 3\alpha + \ldots$$

11. Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

12. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x+1}{2} = \frac{y-2}{4} = \frac{z-2}{5}$ are coplanar, find the equation of the plane containing them.

13. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ at the point (2, 2, –1).

14. Show that the equation

$$2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$$

represents a cone. Find the co-ordinates of the vertex.

15. Find the directional derivative of $xyz - xy^2z^3$ at the point (1, 2, –1) in the direction of the vector $\hat{i} - \hat{j} - 3\hat{k}$.

16. If $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, evaluate $\iint_S \vec{F} \cdot \vec{n} dS$ where S is the surface of the cube bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0, z = 1$.