UG–325 BMS–12/BMC–12

B.Sc. DEGREE EXAMINATION – JUNE 2008.

(BMS–12 : AY 2006–2007 onwards
BMC–12 : AY 2007–2008 onwards)

First Year

Mathematics/Mathematics with Computer Application

TRIGONOMETRY, ANALYTICAL GEOMETRY (3D) AND VECTOR CALCULUS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. If $\frac{\sin x}{x} = \frac{863}{864}$. Find an approximate value of x.

2. If $\tan\frac{x}{2} = \tanh\frac{x}{2}$. Show that $\cos x \cosh x = 1$.

3. Find the angle between the planes $2x - y + z = 3$, $x + y + 2z = 7$.

4. Find k so that the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ may be perpendicular to each other.

5. Find the centre and radius of $2x^2 + 2y^2 + 2z^2 - 4x + 16y + 8z + 20 = 0$.

6. Find the value of a so that the vector $\vec{F} = (z + 3y)\,\hat{i} + (x - 2z)\,\hat{j} + (x + az)\,\hat{k}$ is solenoidal.

7. Find the maximum value of the directional derivative of the function $\phi = 2x^2 + 3y^2 + 5z^2$ at the point $(1,\ 1,\ -4)$.

8. Prove that $\nabla \times \nabla\left(r^n\right) = 0$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. Prove that

$$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10.$$

10. Prove that

$$1 - \frac{1}{2}\cos\theta + \frac{1\cdot 3}{2\cdot 4}\cos 2\theta - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos 3\theta + \ldots\ldots\infty$$

$$= \frac{\cos\theta/4}{\sqrt{2\cos\theta/2}}.$$

11. Find the equation of the plane which passes through the line of intersection of the planes $7x - 4y + 7z + 16 = 0$ and $4x + 3y - 2z + 3 = 0$ and perpendicular to the plane $x - y - 2z + 5 = 0$.

12. Find the equation of the plane which passes through the point $(1, 2, -1)$ and contains the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z+2}{-1}$.

13. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z - 6 = 0$ at $(1, 2, 3)$.

14. Find the equation of the cone which passes through the coordinate axes as well as the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{3} = \frac{y}{-1} = \frac{z}{1}$.

15. A field $\vec{F}$ is of the form $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$. Show that $\vec{F}$ is conservative field and find a function ϕ such that $\vec{F} = \nabla\phi$.

16. (a) Show that $\nabla^2 \left(e^r\right) = e^r + \frac{2}{r} e^r$.

(b) Evaluate $\iiint_V \nabla \cdot \vec{F} \, dV$ where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ bounded by $x = 0, \; y = 0, \; z = 0, \; x = 2, \; y = 1, \; z = 3$.