UG–316 **BMS–04**

B.Sc. DEGREE EXAMINATION – JUNE 2008.

(AY 2005–2006, CY 2006 batches only)

Second Year

MODERN ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Show that the union of two equivalence relations need not be an equivalence relation.

2. Express the following permutation as a product of disjoint cycles :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 3 & 5 & 6 & 7 & 4 \end{pmatrix}.$$

3. Prove that every cyclic group is abelian.

4. Show that the centre H of a group G is a normal subgroup of G.

5. Prove that the intersection of two normal subgroups of a group G is a normal subgroup of G.

6. Show that any homomorphic image of a cyclic group is cyclic.

7. Show that any field F is an integral domain.

8. Prove that the characteristic of an integral domain D is either O or a prime number.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that a non-empty subset H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$.

10. Prove that a subgroup of a cyclic group is cyclic.

11. State and prove Lagrange's theorem on finite group.

12. If a group G has exactly one subgroup H of given order then show that H is a normal subgroup of G.

13. Show that isomorphism is an equivalence relation among groups.

14. State and prove the fundamental theorem of homomorphism.

15. Prove that the set F of all real numbers of the form $a + b\sqrt{2}$ where $a, b \in Q$ is a field under the usual addition and multiplication of real numbers.

16. Prove that any integral domain D can be embedded in a field F and also every element of F can be expressed as a quotient of two elements of D.