UG–319 **BMS–07**

B.Sc. DEGREE EXAMINATION – JUNE 2008.

(AY 2005–2006, CY 2006 batches only)

Third Year

Mathematics

REAL AND COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Show that any countably infinite set is equivalent to a proper subset of itself.

2. Define a complete metric space. Prove that any discrete metric space is complete.

3. If $f : R \to R$ and $g : R \to R$ are both continuous functions on R and if $h : R^2 \to R^2$ is defined by $h(x, y) = (f(x), g(y))$, prove that h is continuous on R^2.

4. If (x_n) is a cauchy sequence in a metric space M and (x_n) has a subsequence (x_{n_k}) converging to x, then show that (x_n) converges to x.

5. If one of $|a|$ and $|b|$ is equal to 1, show that $\left|\frac{a-b}{1-\bar{a}b}\right| = 1$.

6. Prove that the function $f(z) = e^x(\cos y - i \sin y)$ is nowhere differentiable.

7. Evaluate $\int_C \frac{z+2}{z} dz$, where C is the semi circle $z = 2e^{i\theta}, \theta \le \theta \le \pi$.

8. State and prove Cauchy's inequality theorem.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. State and prove Holder's inequality.

10. Prove that any complete metric space is of second category.

11. Let (M_1, d_1) and (M_2, d_2) be two metric spaces. Then show that $f : M_1 \to M_2$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$ for all $A \subseteq M_1$.

12. Prove that a subspace of R is connected if and only if it is an interval.

13. Obtain Cauchy Riemann equations in polar coordinates.

14. Show that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function.

15. State and prove Cauchy's integral formula.

16. Evaluate $\int_0^{2\pi} \frac{d\theta}{1 + a \sin\theta}$, $-1 < a < 1$.