UG–320 **BMS–08**

B.Sc. DEGREE EXAMINATION –
JUNE 2008.

(AY 2005–2006, CY 2006 batches only)

Third Year

Mathematics

LINEAR ALGEBRA AND NUMBER SYSTEM

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Define vector space over a field. Give an example.

2. If $\{u, v, w\}$ is linearly independent in a vector space, prove that $\{u+v,\ u-v,\ u-2v+w\}$ is also linearly independent.

3. Compute the inverse of the matrix $\begin{pmatrix} 2 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$.

4. Verify the Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

5. Find the rank of the matrix $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ 2 & 2 & 2 \end{pmatrix}$.

6. Prove that the number of primes is infinite.

7. Find the reminder when 2^{1000} is divisible by 17 is 1.

8. Prove that for any integer n, $n^5 - n$ is divisible by 30.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. If S is a non empty subset of a vector space V over a field F, prove that

   (a) $L(S)$ is a subspace of V

   (b) $S \subseteq L(S)$

   (c) $L(S)$ is the smallest subspace of V containing S.

10. Explain Gram-Schmidt orthogonalisation process. Using it find the orthonormal basis of $V_3(R)$ with the basis $\{(2, -1, 0), (4, -1, 0), (4, 0, -1)\}$.

11. Define orthogonal complement of a subset S of an inner product space V and prove that it is a subspace.

12. If A and B are two $m \times n$ matrices prove that

(a) $\left(A^T\right)^T = A$

(b) $(A+B)^T = A^T + B^T$.

13. (a) Prove that $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal matrix.

(b) Prove that $\|x+y\| \leq \|x\| + \|y\|$.

14. Find the eigen values and eigen vectors of

$$\begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$$

15. State and prove Wilson's theorem.

16. Prove that $712\,! + 1 \equiv 0 \pmod{719}$.

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