UG–313 **BMS–01**

B.Sc. DEGREE EXAMINATION – JUNE 2008.

(AY 2005–2006 CY 2006 batches only)

First Year

Mathematics

CALCULUS AND CLASSICAL ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. If $y = a\cos 5x + b\sin 5x$ show that $\frac{d^2y}{dx^2} + 25y = 0$.

2. Find the angle between the radius vector and the tangent at any point on the conic section $\frac{l}{r} = 1 + e\cos\theta$.

3. Find the radius of curvature of the curve $xy^2 = a^3 - x^3$ at the point $(a, 0)$.

4. Prove that $\int_a^b f(x)dx = \int_a^b f(t)\,dt$.

5. Show that the sequence $\left\{\frac{1}{n}\right\}$ converges to 0.

6. If $\sum_{n=1}^{\infty} a_n$ is a convergent series then prove that $\lim_{n\to\infty} a_n = 0$.

7. Find the sum of first r coefficients in the expansion of $(1-x)^{-3}$.

8. Find the coefficient of x^n in the expansion of e^{a+bx}.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If $y = x^2 e^x$ show that

$$y_n = \frac{1}{2}n\,(n-1)y_2 - n\,(n-2)y_1 + \frac{1}{2}(n-1)\,(n-2)\,y.$$

10. Find the envelope of $x\cos^3\alpha + y\sin^3\alpha = a$ where α is the parameter.

11. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x dx$.

12. Obtain a Fourier expansion for e^x in the interval $-\pi < x < \pi$.

13. (a) State Leibnitz test for the convergence of an alternating series.

(b) Test the convergence of the series $\sum \frac{(-1)^{n-1}}{n^2}$.

14. Test the convergence of the series

$$\frac{1 \cdot 2}{3 \cdot 4 \cdot 5} + \frac{2 \cdot 3}{4 \cdot 5 \cdot 6} + \frac{3 \cdot 4}{5 \cdot 6 \cdot 7} + \ldots\ldots \infty .$$

15. Find the sum to infinity the series

$$1 + \frac{2}{6} + \frac{2 \cdot 5}{6 \cdot 12} + \frac{2 \cdot 5 \cdot 8}{6 \cdot 12 \cdot 18} + \ldots\ldots \infty$$

16. Find the sum to infinity the series

$$1 + \frac{2^3}{2!} + \frac{3^3}{3\,!} + \frac{4^3}{4\,!} + \ldots\ldots \infty$$