UG–743 **BMS-01**

B.Sc. DEGREE EXAMINATION – JUNE, 2006.

First Year

Mathematics

CALCULUS AND CLASSICAL ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Differentiate $y = (\sin x)^x$.

2. Find the equation of the tangent to the curve $y = \frac{6x}{x^2 - 1}$ at the point $(2,4)$.

3. Find the envelope of the family of straight lines $y = mx + \frac{a}{m}$ for different values of m.

4. Using Bernoulli's formula, find $\int (2x^2 + 1)\cos x dx$.

5. If $u_1 + u_2 + ... + u_n + ...$ is convergent then show that $\lim_{n \to \infty} u_n = 0$.

6. Test the convergence of the series $\sum_{n=0}^{\infty} \frac{n^3 + 1}{2^n + 1}$.

7. Define absolute convergence and conditional convergence.

8. Show that $\sqrt{x^2 + 16} - \sqrt{x^2 + 9} = \frac{7}{2x}$ nearly for sufficiently large values of x.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. (a) If $y = \sin\left(m \sin^{-1} x\right)$, prove that $\left(1 - x^2\right) y_2 - x y_1 + m^2 y = 0$.

(b) Find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$. (7 + 3)

10. (a) What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the print (1,1)?

(b) Find the pedal equation of the curve $r = ae^{\theta \cot \alpha}$. (6 + 4)

11. Obtain a reduction formula for $\int \sin^n x$, where n is a positive integer. Deduce a formula to evaluate $\int_0^{\pi/2} \sin^n x \, dx$. (6 + 4)

12. Express $F(x) = x^2$ as a Fourier series with period 2π, to be valid in the interval $-\pi$ to π. (10)

13. Show that the series $\frac{1}{1^k} + \frac{1}{2^k} + \frac{1}{3^k} + ...$ is convergent when $k > 1$ and divergent when $k \leq 1$. (10)

14. (a) State Leibuitz test for checking the convergence of an alternating series.

(b) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2n - 1}.$$ (3 + 7)

15. Sum the series

$$\frac{1^2}{1!} + \frac{1^2 + 2^2}{2!} + \frac{1^2 + 2^2 + 3^2}{3!} + ... + \frac{1^2 + 2^2 + ... + n^2}{n!} + ...$$ (10)

16. Show that if $x > 0$,

$$\log x = \frac{x - 1}{x + 1} + \frac{1}{2} \frac{x^2 - 1}{(x + 1)^2} + \frac{1}{3} \frac{x^3 - 1}{(x + 1)^3} + ...$$ (10)