UG–744 **BMS-02**

B.Sc. DEGREE EXAMINATION – JUNE, 2006.

First Year

Mathematics

TRIGONOMETRY, ANALYTICAL GEOMETRY OF THREE DIMENSIONS AND VECTOR CALCULUS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Show that

$$\cos 6\theta = 1 - 18\sin^2\theta + 48\sin^4\theta - 32\sin^6\theta .$$

2. Prove that $\cosh^2 x - \sinh^2 x = 1$.

3. Find the real and imaginary parts of Log $(a + ib)$.

4. Find the equation of the line joining the points $(2,3,7)$ and $(2,-5,8)$.

5. Find the centre and radius of the sphere

$$x^2+y^2+z^2-2x-4y+6z-11=0.$$

6. Find the equation of the cone of the second degree which passes through the axes.

7. Find the divergence of $x^2\,\bar{i}+y^2\,\bar{j}+z^2\,\bar{k}$.

8. If $\bar{F}=3x^2y\,\bar{i}+(x^3-3y^2)\,\bar{j}$, compute $\int \bar{F}\cdot d\bar{r}$ along $y^2=4x$ from $(0,0)$ to $(4,4)$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. Prove that $\cos^5\theta\cdot\sin^4\theta=\frac{1}{2^8}[\cos 9\theta+\cos 7\theta-4\cos 5\theta-4\cos 3\theta+6\cos\theta]$.

10. If $\cos(x+iy)=\cos\theta+i\sin\theta$, show that $\cos 2x+\cosh 2y=2$.

11. Sum the series upto n terms:

$$\tan^{-1}\frac{4}{4.1^2+3}+\tan^{-1}\frac{4}{4.2^2+3}+\tan^{-1}\frac{4}{4.3^2+3}+\ldots$$

12. Prove that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-}{2}$; $\frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Find also the point of intersection and the plane through them.

13. Find the shortest distance between the lines $\frac{x+3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Also find the equation of the line of shortest distance.

14. Find the equation of the sphere which passes through the point $(1,-2,3)$ and the circle $z=0$, $x^2+y^2+z^2-9=0$.

15. Find $\nabla\cdot\bar{F}$ and $\nabla\times\bar{F}$ of the vector point function $\bar{F}=xz^3\,\bar{i}-2x^2yz\,\bar{j}+2yz^4\,\bar{k}$ at the point $(1,-1,1)$.

16. Evaluate $\iint_S \bar{F}\cdot\hat{n}\,ds$ where $\bar{F}=4xz\,\bar{i}-y^2\,\bar{j}+yz\,\bar{k}$ and S is the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.

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